FINAL REPORT

on

DEVELOPMENT OF A MATHEMATICAL MODEL OF THE HUMAN OPERATOR'S DECISION-MAKING FUNCTIONS

to

NASA ELECTRONIC RESEARCH CENTER

October 31, 1966

Contract No. NAS 12-37

by

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DEVELOPMENT OF A MATHEMATICAL MODEL OF THE HUMAN OPERATOR'S DECISION-MAKING FUNCTIONS

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INTRODUCTION AND SUMMARY

This is the final report submitted to the National Aeronautics and Space Administration Electronics Research Center in accordance with Contract No. NAS 12-37, Development of a Mathematical Model of the Human Operator's Decision-Making Functions.

This program spanned a period of 16 months, July 1965, through October 1966. The objective of this program was to conduct research leading to the development of a model of the human operator which will advance the state of the art of such modeling. The development of a better human operator model would allow more precise specifications by control systems engineers of input and output equipment which best matches human performance characteristics.

The research described in this report is concerned with the formulation of a mathematical model describing the human operator's decision-making functions in a control system. The model simulates the evolution of control strategies selected by a human operator and the prediction of verbal heuristics used by a human operator. The operator is assumed to be engaged in the on-line control of a dynamic system described by an ordinary linear differential equation subject to initial and final boundary conditions. The operator's task consists of moving the system from the initial state to the terminal state and minimizing a quadratic performance criterion using information concerning state variables and "cost" variables which is obtained from meter readings available at discrete time during the control operation. A summary of the proposed solution and experimental results is presented as follows:

Proposed Solution

To pursue the objective stated in the contract, a mathematical model is developed which attempts to simulate the evolution of the human operator's strategies for the selection of controls on the basis of the observed meter readings. The proposed model consists of four modes of control. They are the heuristic mode, the gradient mode, the terminal mode, and the probing mode. In the heuristic mode, the control strategy consists of selecting controls to maintain invariant relations discovered to exist between successive portions of the task. In the gradient mode, a "cost" reducing control action is applied repeatedly whenever it has been detected. In the terminal mode, the final end-point conditions are approached regardless of sharp increases in the "cost" functional. The probing mode consists of a search procedure and is used whenever the other three modes are not operational.

The development of the heuristic mode of control is central in this research. The mathematical model attempts to discover dimensionless parameters relevant to the objective functional, and attempts to maintain these parameters at appropriate levels. A heuristic resulting from this procedure is exemplified by the verbal statement: "In order to minimize 'cost', choose controls so that the ratio of the reading on meter 3 to the reading on meter 5 is equal to 10."

The dynamic system used in the simulation study is described by a first-order or second-order differential equation subject to certain specified boundary conditions. The control signals are selected from a predetermined set of values.

Experimental Investigations

The approach described above was investigated experimentally by allowing 14 subjects to solve 23 first-order control problems (Mark I model), and allowing 14 additional subjects to solve 12 second-order control problems (Mark II model). The subjects were tested in an on-line, real-time environment with their control selections fed into a Control Data 3400 computer. The computer used the selected values to up-date the values of the meter-readings. These values were then printed out and displayed to the subjects who were then required to make their next selection. For the Mark I experiments the number of selections per problem varied between 8 and 38; for Mark II the number of selections was 20 for each problem. The problems permitted the use of five control selections: $y = -2, -1, 0, 1,$ or 2. The number of meter-readings displayed to the subjects was six and eight for Mark I and Mark II, respectively. The subject was asked to minimize the consumption of "fuel", and to hit target.

At the end of each problem, the subjects were asked to state their recommendations to a second hypothetical controller who would soon be required to solve a similar set of problems. These statements were recorded on tape and were later used to identify heuristics used by the subjects. Exclusive of the time for instruction, approximately 1-1/2 hours were required for each subject to finish the set of problems. These same problems were also solved by the Mark I and Mark II computer simulation models.

Simulation and Experimental Results

The results from the computer simulation and those from tests of the subjects were analyzed in two respects: (1) performance as measured by total fuel consumed for each problem and (2) the agreement between the heuristics used by the subjects and those used by the simulation model.

For the Mark I experiment a high linear correlation (r = 0.916) was obtained between the subject median "fuel" consumption and the model "fuel" consumption. However, percentage deviations in excess of 100 percent were obtained for three of the problems. Learning curves were fitted to the ratio of the Mark I model's "fuel" consumption to the subject median "fuel" consumption. The curves showed improving subject performance relative to the model between Problems 5 and 14, at which point, as expected, subject performance degraded sharply because of increased difficulty in the problems. Subject learning again took place between Problems 14 and 23.

A three-man panel was used to analyze the taped statements made by the subjects to determine the number of subjects using the same heuristic as that used by the Mark I model. From Problem 8 to the end of the set, the panel agreed that at least 11 of the 14 subjects used the same heuristic as the model. The average conditional probability of a correctly matched heuristic, given the model's heuristic, was found to be 0.83 over the last 15 problems.

An additional analysis was made by comparing the performance of the subjects with computed results obtained from the use of eight hypothetical strategies. The correlation coefficients for the subject median costs and these eight strategies varied between 0.268 for a random selection of controls to 0.918 for a composite strategy expected to yield a maximum correlation.

The results of the Mark II computer simulation showed good agreement in terms of performance for small values of the time constant. The agreement degraded markedly as the time constant was increased. A learning curve was fitted to the last six of the 12 control problems. The learning curve was of the same type as used to analyze the Mark I results.

The same three-man panel was used to analyze the verbal statements. The average conditional probability of a matched heuristic, given the model's heuristic was found to be 0.15; the unconditional probability that a subject's heuristic will match some heuristic in the simulation list of heuristics was approximately 0.50. These small probabilities are attributed to unappropriate choices of the parameters in the Mark II model.

Evaluation of Results

Only one model is proposed in this study and data are gathered which tend to support it. It does not follow that this model is validated. Other models may be equally consistent with the data. A "modern" scientific approach would formulate competing models as alternative hypotheses and conduct an experi- of sufficient precision to be capable of rejecting all but one of these hypotheses. At present, however, there appears to be a scarcity of mathematical structures that can be used to evolve the verbal heuristics of the human controller.

It is believed that the instructions to the subject, the conduct of the experiments, and the use of a panel of judges have yielded reliable results. It is true that no other investigator can use the same judges or subjects. Nevertheless, it is predicted that if he will conduct a similar experiment, using the same number of subjects, the same number of judges, and the same methods of analysis, he will arrive at conclusions that are in statistical agreement with those obtained in this study.

It is concluded from this research that the Mark I model offers a feasible approach to modeling human decision-making in first-order control systems of the type investigated. In addition to producing the heuristics used by human controllers with high probability, it also gives high correlations with human performance curves.

For second-order control systems the agreement between the Mark II model and the subjects was not impressive. Much of this difficulty is believed to be the result of making inappropriate parameter assignments in the model. However, the results obtained suggest that, even with appropriate assignments, the model may fail for large values of the time constant.

AN OVERVIEW OF THE RESEARCH

The following sections present an overview of the research project. Some earlier work in modeling of human decision-making processes in a control task is reviewed. The evolution of the project is outlined. The concept of dimensional analysis is introduced. Some basic assumptions are made in the characterization of the human controller. This section concludes with a brief description of the proposed mathematical model.

Review of Previous Research

Previous work in the study of manual control from the engineering point of view dates back to 1947 when Tustin(1) proposed a description of the operator's response and its implications for controller design. In 1948, Ragazzini(2) discussed engineering aspects of the human as a servomechanism. Since that time, research interest in the mathematical characterization of the human operator in the control system has greatly intensified. Papers and reports describing these models exist in abundance. Practically all of these proposed models attempt to describe major characteristics of the human operator in the form of a transfer function. The models are primarily developed to describe compensatory tracking behavior and pursuit tracking behavior. Although both tracking behavior and decision-making behavior are regarded as major characteristics of the human in a manual control system, little work has been done on the mathematical modeling of the decision-making behavior of the human operator in a control system. The fact that research in mathematical modeling of human decision-making lags behind that of human tracking behavior is primarily due to the difficulty of obtaining mathematical descriptions of decision-making behavior. Physical laws may be used to characterize the tracking task but not the decision-making behavior since the latter involves a mental process which deals with such aspects of thinking, experience, extrapolation, judgment, inference, and generalization.

In 1962 Thomas(3) developed a set of test problems that could serve as a useful tool in studying the characteristics of human controllers. The optimal solutions to the test problems were derived using dynamic programming(4,5,6) or the maximum principle(7). It was proposed that human subjects be repeatedly allowed to generate solutions to these problems in order to determine whether or not human subjects could "learn" optimal control by repeated trials. The only information given to the subject would consist of the values of the state variables and the value of the objective functional after the completion of a trajectory. The performance obtained from the mathematical solution to the control problem could then be compared with subject performance.

In the dissertation of Ray(8), the proposed approach was carried out for one of the control problems. In general, it was found that about half of the subjects tested achieved nearly optimal control in approximately 20 repetitions of the problem. A stochastic control problem was similarly tested by Rapoport(9). It should be emphasized that none of these investigations yields a mathematical model of how the human organizes his previous experience in order to improve his performance. In particular, these investigations can not be related to the question of whether the human controller can be characterized as a Bayesian decision-maker as studied by Edwards[10]. Moreover, these investigations avoid the difficulties associated with obtaining quantitative characterizations of concept evolution, generalization, and judgment. Some of these difficulties are made explicit by Watanabe[11].

Evolution of the Project

The aim of the project is incorporated in the following statement:

> "Using experimental procedures, investigate the role of higher mental processes such as those involved in judgment, extrapolation from the knowledge of immediately previous performance and similar human capabilities as they influence man's total performance in manual control systems."

Although this aim is directed toward the use of experimental procedures, it is clearly necessary to develop a theoretical basis for the experimentation. The initial theoretical basis took the form of dynamic programming. In general, it was asked whether the decision-making processes of the human controller could be represented by an algorithm based upon dynamic programming.

It was concluded that systematic recursive structures of the type associated with dynamic programming may not be a good way to characterize human reasoning processes. Such structures do not permit a human controller to revise his guesses, do not permit him to introduce external information at stages after the first stage, and do not lend themselves to the use of heuristics which evolve from stage to stage. The use of policy space, rather then function space, appears to be more typical of the human decision-maker. However, because of the loss of the Markov property, a more general version of approximation in policy space would be needed than that given by Howard[5].

In order to include subjective estimates of the decision-making algorithm, it appeared necessary to use elements of Bayesian statistics because these methods alone appear to deal with the problem of combining in a mathematical way measures of belief and actual observations. In the context of the present aim, it appeared desirable to apply these methods directly to policies, rather than to the estimates of parameters in models as is typical Bayesian problems.

Based on these ideas some attempts were made to formulate the decision-making process in terms of policies and Bayesian probabilities. In particular, the Bayesian probability was taken to be the probability that the policy used at a particular trial would optimize the performance criterion. The succession of trials would then serve to support, or deny, the optimality of any tested policy. By regarding such trials as successes or failures for a given policy, the probability that a given policy will generate optimal control could be updated using a Bayesian method.

Further development of these concepts showed a strong presumption of an environment in which "repeated trials" could take place. The "learning" of the human controller was thus considered as the result of repeated trials under nominally identical conditions. However, in the present context, it is clearly preferable to assume that the human does not have the luxury of repeated trials that will facilitate learning. In the space environment, for example, as astronaut does not have an opportunity to generate many different trajectories in order to find a minimum-fuel trajectory. In practice there is only one trajectory generated by the astronaut. Consequently, any "learning" by the astronaut must be accomplished during the generation of the trajectory. Such on-line learning may permit him to improve his performance during later stages of the trajectory.

Several assumptions were then made about the human controller:

(1) The human controller will "review" his accumulated control experience from time-to-time in order to extract the information which would be likely to increase performance during the remainder of the trajectory.

(2) The review times occur at the discretion of the controller.

(3) The controller will develop his trajectory in segments. In generating a trajectory between A, B, and C, the human controller may first restrict his attention to the problem of generating a trajectory between A and B, or even on initial portion of the trajectory between A and B.

(4) When his accumulated experience is evaluated at a review time and the extrapolation of his current control efforts are judged by the controller to be nonoptimal relative to some alternative control efforts, then he may abandon his current control effort and begin a new, but "similar", control problem. The problem is a new one in the sense that the initial point of the desired trajectory now coincides with the current location of the controller. The final point is not changed and the form of the performance criterion is not changed, but the form applies only to the remaining portions of the trajectory.

Thus, the controller is envisioned as generating a sequence of similar control problems each one of which may be partially completed. The final trajectory is considered to be made up of these subtrajectories.

These developments led quite naturally to the present form of the model. The emphasis in the above assumptions on the similarity of the subtrajectories forced a distinction to be made between the control values actually selected and the verbal description of the kind of similarity that existed between the subtrajectories. The verbal description could remain identical and yet the associated control actions could be quite different. In fact, it was now convenient to label as "heuristics" the verbal statements of similarities when translated into recommendations for choosing controls. In simpler terms, human control strategy consists of two parts: (1) operational control actions and (2) verbal statements of how to choose these controls.

The mathematical modeling problem was also made easier by the recognition of the role of "similarity" between subtrajectories. The kind of mathematical analysis customarily associated with similar physical processes has its roots in dimensional analysis.[(13)] Dimensional analysis, in turn, furnishes a very primitive and general approach to problems of finding empirical relations among physical variables. The gathering of data suitable for detecting invariant relations by dimensional analysis appears to be ideally suited to the evolution of heuristics. That is, as more data are analyzed, the verbal description of whatever invariance is found can be translated into a prescription of how to choose controls to take advantage of the structure thus found. The generality of the verbal statements and the conviction of the controller in their validity depend upon the experience of the controller. In summary, it was concluded that a mathematical system was needed which could extract the maximum of generality from empirical data, and could evolve and yield verbal statements of invariant structure which could serve as a basis for control selection.

Basic Assumptions of the Model

In a general form, decision-making by a human controller can be regarded as a sequence of decisions in which each decision consists of selecting one control from a set of possible controls. This approach usually yields probabilistic decision models with the selection of various controls governed by conditional probabilities. In practice, because of large numbers of possible control sequences and complicated dependency relations, the required joint probability distributions are often exceedingly difficult to determine.

The choice of individual controls is not the concern of this research. Instead, we are concerned with the evolution of rules, called heuristics, used by controllers to select controls. As a simple analogy, we do not ask what move a particular player will make next in a chess game. Instead, we ask what strategy, if any, has the player evolved? It is clear that if it is known that the player's strategy is to gain control of the center of the board, then the number of possible moves consistent with this strategy may be considerably reduced. Different sequences of moves may be consistent with the same strategy, and these may sometimes be regarded as equivalent, even though the individual moves involved are quite different. This may yield an appreciable simplification of probabilistic models of his sequence of moves.

The class of control problems considered in this report are assumed to be associated with the control of physical variables which have dimensions of mass, length, time, and temperature. As a particular example, we have considered the task of generating minimum-fuel trajectories. The variables associated with such a task may include position, velocity, acceleration, orientation angles, thrust, ambient temperature, fuel consumption rate, etc.

Besides restricting attention to control problems involving physical variables, we have also assumed that the human controller extracts his information entirely from a set of meters which measure these physical variables. In particular, it is assumed that one meter displays the "cost" associated with each decision interval. It is implicitly assumed that the remaining meters measure variables whose values are relevant to the control problem. If this assumption is not true, then the model described in this research would be expected to remain in a state of search and would predict that no heuristic would be evolved by a human controller.

Because the model involves the evolution of heuristics by a human controller, it is required that the control problems of interest be of sufficiently long duration so that the human has time to search, to observe relationships among his meter readings, and to "learn" in an on-line, real-time environment. This requirement further restricts the class of control problems to which this research is applicable.

Concept of Dimensional Analysis

The basic principles of dimensional analysis are given by Pankhurst[12]. The primary reason for the assumption of meter readings of dimensional quantities is the fact that a primitive mathematical basis, derived from dimensional analysis, may then be used to yield a set of possible heuristics. The Pi Theorem is significant in that it restricts the number of combinations of the variables needed in a given control problem. In a typical case these combinations constitute a new, and usually smaller, set of variables in which each variable is dimensionless. For a dimensionless variable, any change of scale for a unit of mass, length, time, or temperature will not alter its magnitude.

We next state more explicitly the mathematical structure of dimensional analysis.

Let x_i, $i = 1,\ldots,n$, be a positive numerical magnitude for the physical quantity X_i when x_i is expressed in terms of a set of reference units U_j, $j = 1,\ldots,m$. By the process of changing units, a new set of units U_j', $j = 1,\ldots,m$ may be used where

$$u_j' = u_j/\tau_j \ , \quad (j = 1,\ldots,m) \quad , \tag{1}$$

and τ_j denotes a positive real number which is dimensionless. Under such a change of units, x_i is changed to x_i' where

$$x_i' = \tau_i^{a_{i1}} \ldots \tau_m^{a_{im}} x_i \quad (i = 1,\ldots,n) \quad . \tag{2}$$

The real numbers, $(a_{i1},\ldots,a_{im})$, are called the dimensions of X_i. The associated n x m matrix, $\underline{A} = (a_{ij})$, consisting of the rows, $(a_{i1},\ldots,a_{im})$, is called the dimensional matrix for the set of physical quantities $\{X_i\}$. Whenever any row of $\underline{A}$ consists entirely of zeros the corresponding physical quantity is said to be dimensionless; that is, $a_{ij} = 0$, $j = 1,\ldots,m$ implies X_i is dimensionless.

Assume that a physical quantity X has a numericaly magnitude x which is expressible as an equation $x = f(x_1,\ldots,x_n)$ which is valid under an arbitrary change of units given by (1) so that $x' = f(x_1',\ldots,x_n')$. From (2) it follows that

$$f(\tau_1^{a_{11}}\ldots\tau_m^{a_{1m}}x_1,\ldots,\tau_1^{a_{n1}}\ldots\tau_n^{a_{nm}}x_n) = \tau_1^{a_1}\ldots\tau_m^{a_m}f(x_1,\ldots,x_n)$$

where a_j, $j = 1,\ldots,m$ denote the dimensions of X. Functions which satisfy such a relation identically for all postive τ's are said to be dimensionally homogeneous with respect to the m reference units, $U_1,\ldots,U_m$. In particular, if $f(x_1,\ldots,x_n) = 0$ and if $f(\underline{x})$ is dimensionally homogeneous, then the equation

$$f(\tau_1^{a_{11}}\ldots\tau_m^{a_{1m}}x_1,\ldots,\tau_1^{a_{n1}}\ldots\tau_m^{a_{nm}}x_n) = 0$$

is an identity in τ_j, $j = 1,\ldots,m$.

The basic theorem of dimensional analysis is Buckingham's[13] Pi Theorem. The specific form of the Pi Theorem as proved by Brand[14] is stated as follows:

Buckingham's Pi Theorem -- Let physical quantities X_i, $i = 1,\ldots,n$, have the dimensional matrix of rank $r = n - k$:

$$\underline{A} = \begin{bmatrix} \underline{P} & \underline{R} \\ \underline{Q} & \underline{S} \end{bmatrix}$$

where $\underline{P}$ is a non-singular r x r matrix. Let $f(x_1,\ldots,x_n)$ be dimensionally homogeneous with respect to m reference units $U_1,\ldots,U_m$. Then the equation

$$f(x_1,\ldots,x_n) = 0$$

is equivalent to

$$f(1,\ldots,1,\pi_1,\ldots,\pi_k) = 0$$

in which the first r arguments are 1, and

$$\pi_i = x_1^{e_{i1}} \ldots x_n^{e_{in}} \qquad (i = 1,\ldots,k)$$

are $k = n - r$ independent and dimensionless quantities with the $k \times n$ matrix of exponents given by

$$\underline{E} = (\underline{QP}^{-1}, \underline{I}_k)$$

with $\underline{I}_k$ denoting the $k \times k$ unit matrix.

Suppose that the X_i, $i = 1,\ldots,n$, are physical quantities whose numerical magnitudes x_i are displayed on n meters. By the Pi Theorem any dimensionally homogeneous functional relation among the magnitudes of these physical quantities is equivalent to a functional relation among dimensionless products and ratios of these magnitudes. We form these $(n - r)$ dimensionless variables, and associate a possible heuristic with the invariance of each of them. With the additional possible invariance of each of the n meters, it follows that at most $(2n - r)$ invariants are associated with a given control problem. These give rise to a set of $(2n - r)$ possible heuristics. A list of these heuristics can be made as soon as the dimensions of the quantities displayed on the meters are known.

Basic Assumptions for the Human Controller

It is clear that psychological differences, as well as differences in experience and training may yield widely different behavior among human controllers. A controller of little experience may only discover that a certain meter reading is constant whenever the cost meter reverses a downward trend. A more experienced controller may discover that a certain combination of meter readings is equal to a constant whenever the cost meter reverses a downward trend. A still more experiences controller may discover a functional relation that exists between two or more meter readings when the cost meter reverses a downward trend.

The amount of training, the kind of training, the familiarity with physical laws, etc., are clearly important in determining the complexity of the invariants that may be detected by a human controller. A controller for which the distinction between velocity and acceleration is not clear will probably not detect complex invariants. On the other hand, a highly trained individual who is familiar with coordinate systems, positions, velocities, accelerations, inertia, drag, Newton's Laws, etc., may detect high levels of complexity.

For the purpose of computer simulation it is assumed that a human controller will eventually note the invariance of any meter reading, or appropriate combination of meter readings, that occurs when his objective functional is optimized over individual decision intervals. The invariance, discovered as a result of this suboptimization, will be expressed as a verbal heuristic.

In the simplest control problems, or with a remarkably appropriate meter, the invariance of a single meter may be taken as the basis of a heuristic. As a hypothetical example: "In order to minimize costs, choose controls so that meter three reads 10". In more complex cases, the invariance of ratios or products of meter readings may be discovered and used. For example: "In order to minimize costs, choose the controls so that the ratio of the reading on on meter three to that on meter four is equal to 10." As noted above, it is conceptually possible that even functional relations among dimensionless combinations of the meter readings could be detected and used as a basis for heuristics. However, in the present study it is assumed that the invariance of the readings on single meters, or dimensionless combinations of meter readings, provides a sufficiently "rich" formulation for all but the most experienced controllers.

We also note that no assumptions are made about the manner in which the human arrives at his heuristics. It is merely postulated that he obtains his heuristics by inductive generalization from empirical data. The purpose of the computer simulation model is to predict the human controller's heuristic. It is not supposed that the human controller really processes his data in the form indicated by the concepts of dimensional analysis. It is merely asserted that such a model can yield heuristics. The main question is whether there is agreement between the heuristics obtained from the model and those obtained from a human controller.

One method of obtaining knowledge of a controller's strategy is to ask him to verbalize it. His response may be quite misleading. Some controllers may "invent" a strategy because the question suggests they are "supposed" to have one; some controllers may be deceptive; some controllers may lie; some controllers may be unable to verbalize; some controllers may be inhibited from admitting that they are "experimenting" with alternative strategies; some may be reluctant to verbalize changes from one strategy to another; some may be reluctant to verbalize "irrational" behavior; and some may be reluctant to verbalize "rational" behavior. Because of these problems and others there is a general reluctance among many psychologists to deal directly with verbal data. Despite these well-known difficulties and a historical precedent against it, this study focuses attention on verbal data.

The verbal statement of a heuristic is simply a grammatical statement in which it is recommended that controls be slected so that an observed invariance among meter readings can be attained or maintained. Clearly, there are many equivalent ways in which a heuristic can be verbalized. In this study the computer simulation model is programmed to print out a particular verbal form for each of the possible heuristics. There remains the unavoidably subjective problem of deciding whether a controller's statements conform to the model's predicted verbal heuristic. This analysis is discussed in more detail in later sections.

In complex cases, the invariants that exist in a given control task may change during the course of the task. In accord with the basic assumption, this would require the human controller to change heuristics. Moreover, invariants that are mutually consistent at one time may be inconsistent at a later time. In this case the human controller may choose to maintain one invariant relation at the expense of another, and thereby let one heuristic dominate at the expense of another. Alternatively, the human controller may attempt to compromise and maintain several invariants approximately near their desired levels. In this case he attempts the simultaneous use of competitive heuristics. As difficulties develop, he may abandon previous heuristics and search for new ones, etc. In the present research several of these difficulties are avoided by assigning priorities to the possible heuristics. If several heuristics are available, that heuristic having the highest priority is used.

As a guide to the assignment of priorities, it is assumed that the human controller will first seek an individual meter that reads a constant value whenever the cost meter reverses a downward trend. If no single meter is found to read a constant value when the cost meter reverses a downward trend, then the controller will examine certain combinations of meter readings for invariance. In particular, ratios of distances, ratios of velocities and other "physically meaningful" combinations of the meter readings will be examined.

Description of the Mathematical Model

Several complexities are introduced in the process of developing a a mathematical model capable of generating the trajectories demanded of human controllers and capable of predicting the verbal heuristics that human controllers will use. We have attempted to keep this model as simple as possible. Even so, it was found necessary to introduce four control modes: (1) probing mode, (2) gradient mode, (3) heuristic mode, and (4) terminal mode. Only the heuristic mode is associated with the use of an invariant as a heuristic, the gradient mode of control is used whenever a cost reducing control has been discovered, the terminal mode of control is used when the final point of the trajectory is to be obtained regardless of cost, and the probing mode of control is used whenever the other three modes are not operational. In the proposed model, simple computer logic for switching from one mode to another is employed. The proposed four-mode control scheme was designed to generate data that could be subsequently analyzed for invariant relations.

In descriptive terms the mathematical model performs as follows. Suppose a particular control has just yielded a decreasing cost increment as associated with the current decision interval. Under the gradient mode this control choice will be made repeatedly until the cost increment increases. When the increase occurs, a quadratic interpolation is used over the three most recent cost increments in order to estimate the time at which the minimum actually occurred. Every meter reading is then linearly interpolated to obtain estimated meter readings, at the time the minimum occurred. These meter readings, and the appropriate dimensionless combinations of them, are stored as the first row of the matrix. After several minima have occurred, the computer program examines each column of the matrix to determine whether any meter, or combination of meters, is approximately constant. For each invariant thus found, a heuristic is identified. The mathematical model then enters the heuristic mode of control, and that heuristic is used having the highest pre-assigned priority as a basis for selecting controls. If no invariants are found, the program typically reverts to the probing mode.

A simplified flow chart of the computer simulation of the model is given in the the next section. A complete listing of the program instructions is given in Appendix A.

MATHEMATICAL STRUCTURE OF CONTROL PROBLEMS

In the following sections the mathematical structure of Mark I and Mark II models is discussed. First-order control problems are used in the Mark I model, and second-order control problems are studied in the Mark II model. The criterion of control is to minimize the "fuel" consumption required to move the system from an initial state to a prespecified terminal state. The choice of control is limited within a predetermined set.

Transformation of Equations in Mark I Model

The first-order control problems studied in this research consist of changing an initial velocity v_0 to a final velocity v_f over N discrete time-intervals so that the total "fuel" consumption (cost) is minimized. At each time interval a control y_{k+1} is selected from the set $\{-2, -1, 0, 1, 2\}$ and applied to the previous velocity v_k to yield a new current velocity v_{k+1} in accordance with the recursive relation

$$v_{k+1} = a\, v_k + b\, y_{k+1} \,, \qquad k = 0,\ldots,N-1 \quad ,$$

where a and b are given constants. The performance criterion which describes the "fuel" consumption is given by

$$C = A \sum_{k=1}^{N} (v_k - V)^2 + B(v_N - v_f)^2 \quad ,$$

where A, B, and the reference velocity V are given constants.

An examination of the objective functional shows that the cost increments associated with each decision are given by $A(v_k - V)^2$. These cost increments may be minimized by bringing the system velocity v_k as close as possible to the reference velocity V. With N sufficiently large, a suboptimizing procedure of the following form is obtained: Choose controls so that the cost increments are minimized as soon as possible and maintain the cost increments at their minimum levels as long as possible before attempting to achieve the desired final velocity.

The existence of the reference velocity is not known to the subjects, and is not used in the computer simulation logic. For the subjects it is assumed that they will discover the existence of the reference velocity and choose their controls in approximate accord with the above suboptimization procedure. The computer simulation logic is structured so that under a gradient mode of control and a heuristic mode of control the cost increments are minimized. The probing mode of control and the terminal mode of control do not minimize cost increments and do not strictly conform to the above suboptimizing procedure. The structure of the computer logic is discussed in more detail later.

In the Mark I model it is assumed that the following information is displayed on six meters:

Meter Number	Variable Displayed
1	current control choice, y
2	number of decisions remaining, d
3	current velocity, v
4	difference between desired final velocity and current velocity, Δv
5	fuel consumption (cost) incurred during current decision interval, ΔC
6	current cumulative fuel consumption (cumulative cost), C

The last five variables are updated after each control choice in accordance with the following equations:

$$d_c = d_p - 1$$

$$v_c = av_p + by_c$$

$$\Delta v_c = v_f - v_c$$

$$\Delta C_c = k(v_c - V)^2$$

$$C_c = C_p + \Delta C_c \quad ,$$

where the subscripts p, c, and f denote previous, current, and final values, respectively, and V denotes the reference velocity.

Derivation of possible heuristics is explained as follows. In a mass-length-time-temperature (MLTθ) system of units, the control variable is assumed to have dimensions of length, (0,1,0,0); the number of decisions remaining has dimensions of time, (0,0,1,0); the current velocity and velocity difference have units (0,1,-1,0); and the "fuel" consumption is measured in pounds and has units of (1,1,-2,0). Thus, the dimensional matrix associated with these meters is given by:

$$\underline{A} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} M & L & T & \theta \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix} \end{array} \quad .$$

Using the notation described in the preceding section, we form the following rearranged, nonsingular, submatrix of $\underline{A}$:

$$\underline{P} = \begin{array}{c} \\ 2 \\ 4 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccc} L & T & M \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \end{array} ,$$

and a residual matrix $\underline{Q}$:

$$\underline{Q} = \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} L & T & M \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \end{array} .$$

Using these matrices we find that

$$P^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

and

$$\underline{E} = (-\underline{Q}\ \underline{P}^{-1}, \underline{I}_3) = \begin{array}{c} \begin{array}{cccccc} 2 & 4 & 6 & 1 & 3 & 5 \end{array} \\ \begin{bmatrix} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array} ,$$

and rearranging the columns to correspond to a natural ordering of meters we have

$$\underline{E} = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} .$$

Thus, there are $n - r = 6 - 3 = 3$ combinations of meters which give dimensionless combinations of the meter readings. From the $\underline{E}$-matrix it is seen that these combinations are given by

$$\pi_7 = m_1/(m_2\ m_4) = y/(d\Delta v)$$

$$\pi_8 = m_3/m_4 = v/\Delta v$$

$$\pi_9 = m_5/m_6 = \Delta C/C \quad ,$$

where m_j denotes the magnitude of the variable which appears on meter j, $j = 1,\ldots,6$. The nine measures, associated with meters one through six and the three combinations of meter readings, constitute the basic numerical measures used in the Mark I model. The invariance of any one of these measures yields a possible heuristic.

Table 1a shows a list of the nine possible heuristics for the Mark I control problems. Statements one through six correspond to the invariance of meters one through six respectively. Statements seven, eight, and nine correspond to the invariance of the dimensionless parameters, π_7, π_8, and π_9, respectively. The derivations given above show that these statements depend only on the meters available to the controller, and on the dimensions associated with the variables measured by the meters. The table also shows the pre-asigned priorities assigned to these possible heuristics.

TABLE 1a. LIST OF POSSIBLE HEURISTICS FOR MARK I CONTROL PROBLEMS

Number	Priority	Statement
1.	3	To minimize fuel consumption, choose a certain control repeatedly.
2.	9	To minimize fuel consumption, choose controls so that the number of decisions remaining is held equal to a certain constant.
3.	1	To minimize fuel consumption, choose controls so that a certain velocity is maintained.
4.	2	To minimize fuel consumption, choose controls so that the difference between the current velocity and the final velocity is held equal to a certain constant.
5.	7	To minimize fuel consumption, choose controls so that the fuel consumption associated with each decision interval is minimized.
6.	8	To minimize fuel consumption, choose controls so that the cumulated fuel cost is held equal to a certain constant.
7.	5	To minimize fuel consumption, choose controls so that the control value divided by the product of the number of decisions remaining and the difference between the current velocity and final velocity is held equal to a certain constant.
8.	4	To minimize fuel consumption, choose controls so that the current velocity divided by the difference between the current velocity and final velocity is held equal to a certain constant.
9.	6	To minimize fuel consumption, choose controls so that the fuel consumption associated with each decision interval divided by the cumulated fuel consumption is held equal to a certain constant.

Transformation Equations in Mark II Model

The second-order control problems studied in this research consist of changing an initial position u_0 to a final position u_f over N discrete time intervals so that the total fuel consumption (cost) is minimized. The dynamic structure is obtained from $m\ddot{u} + c\dot{u} = Ky$ by replacing $\dot{u}$ and $\ddot{u}$ by $(u_k - u_{k-1})/T$ and $(u_k - 2u_{k-1} + u_{k-2})/T^2$, respectively, where T denotes the time interval between control selections. With T set equal to 1, this replacement yields

$$v_k = \Delta u_k = u_k - u_{k-1} = (\Delta u_{k-1} + \eta\, y_k)/(1 + \xi) \quad ,$$

where $\xi = (c/m)$ and $\eta = K/m$, and $k = 1,\ldots,N$. Three different values of ξ were used. These values were 0.4140, 0.0908, and 0.0444 so that the time constants, $\tau = 1/\xi$, are given by 2.4, 11.0, and 22.5, respectively. These small, medium, and large values of the time constant yield system responses to the operator's control values that are fast, medium, and slow, respectively. These responses correspond to the relative importance of the drag and inertia terms. The values of η were set equal to 1.0 for all of the Mark II control problems.

As in the case of the Mark I control problems, the control selections are made from the set $\{-2, -1, 0, 1, 2\}$. At each time interval a control y_{k+1} is selected and the resulting velocity is obtained from the expression:

$$v_{k+1} = (\Delta u_k + \eta\, y_{k+1})/(1 + \xi) \quad .$$

The new position u_{k+1} is then computed as follows:

$$u_{k+1} = u_k + v_{k+1}, \quad k = 0,1,\ldots,N-1 \quad .$$

The "fuel" consumption is measured by

$$C = A \sum_{k=1}^{N} (v_k - V)^2 + B(v_N + v_f)^2 \quad ,$$

where A, B, and the reference velocity V are given constants.

In exactly the same manner as the Mark I control problems, the existence of the reference velocity is assumed to be unknown to the subject, and is not used in the computer simulation logic.

In the Mark II model it is assumed that the following information is displayed on 8 meters:

Meter Number	Variable Displayed
1	current control choice, y
2	number of decisions remaining, d
3	current position, u
4	difference between final position and current position, Δu
5	fuel consumption (cost) incurred during current decision interval, ΔC
6	current cumulative fuel consumption (cumulative cost), C
7	current velocity, v
8	current acceleration, w .

Variables two through eight are updated after each control choice in accordance with the following equations:

$$d_c = d_p - 1$$

$$v_c = (v_p + \eta\, y_c)/(1 + \xi)$$

$$u_c = u_p + v_c$$

$$\Delta u_c = u_f - u_c$$

$$\Delta C_c = K(v_c - V)^2$$

$$C_c = C_p + \Delta C_c$$

$$w_c = v_c - v_p \quad .$$

where the subscripts p, c, and f denote preceding, current, and final values, respectively.

The possible heuristics are derived as follows. An assignment of dimensions analogous to that used for Mark I yields the following dimensional matrix for the eight meters of the Mark II problems:

$$\underline{A} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} M & L & T & \theta \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix} \end{array} \quad .$$

A rearranged, nonsingular, submatrix of $\underline{A}$ is given by

$$
\underline{P} = \begin{array}{c} \\ 2 \\ 7 \\ 6 \end{array}
\begin{array}{c}
\begin{array}{ccc} L & T & M \end{array} \\
\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}
\end{array} ,
$$

with the residual sub-matrix of $\underline{A}$ given by

$$
\underline{Q} = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ 5 \\ 8 \end{array}
\begin{array}{c}
\begin{array}{ccc} L & T & M \end{array} \\
\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & 0 \end{bmatrix}
\end{array} .
$$

The final $\underline{E}$-matrix is found to be given by

$$
\underline{E} = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix} .
$$

Because there are five rows of the $\underline{E}$-matrix there are five combinations of meter readings which are dimensionless. From the rows of the $\underline{E}$-matrix these combinations are seen to be given by

$$\pi_9 = m_1/(m_2\ m_7) = y/(dv)$$

$$\pi_{10} = m_3/(m_2\ m_7) = u/(dv)$$

$$\pi_{11} = m_4/(m_2\ m_7) = \Delta u/(dv)$$

$$\pi_{12} = m_5/m_6 = \Delta C/C$$

$$\pi_{13} = (m_2\ m_8)/m_7 = dw/v \quad .$$

These five combinations represent 3 distance ratios $(\pi_9, \pi_{10}, \pi_{11})$, a cost ratio $(\Delta C/C)$, and a velocity ratio (dw/v). The measures obtained from the eight meter readings and the five dimensionless combinations of meter readings constitute the basic numerical measures used in the Mark II model. The invariance of any of these measures gives rise to a possible heuristic.

Table 1b shows a list of 13 possible heuristics for the Mark II control problem. The first eight statements correspond to the invariance of the individual meter readings; the last five statements correspond to the invariance of the above dimensionless combinations. The table also shows the assigned priorities associated with those possible heuristics.

FLOW CHARTS AND PROGRAMMING PROCEDURES

In the following sections the computer simulation program is discussed in a simplified form. A complete listing of the FORTRAN instructions is given in Appendix A for the Mark I and Mark II models. In addition, as a typical example, the simulation output for subtrajectory ten is discussed in some detail. This includes an account of the control mode used at the time of each control selection, the switching logic for changing from one mode to another,

TABLE 1b. LIST OF POSSIBLE HEURISTICS FOR MARK II CONTROL PROBLEMS

Number	Priority	Statement
1.	5	To minimize fuel consumption, choose a certain control repeatedly.
2.	8	To minimize fuel consumption, choose controls so that the number of decisions remaining is held equal to a certain constant.
3.	2	To minimize fuel consumption, choose controls so that the current position is held equal to a certain constant.
4.	3	To minimize fuel consumption, choose controls so that the difference between the current position and the desired final position is held equal to a constant.
5.	6	To minimize fuel consumption, choose controls so that the fuel consumption associated with each decision interval is minimized.
6.	7	To minimize fuel consumption, choose controls so that the current cumulated fuel cost is held equal to a certain constant.
7.	1	To minimize fuel consumption, choose controls so that a certain velocity is maintained.
8.	4	To minimize fuel consumption, choose controls so that a certain acceleration is maintained.
9.	9	To minimize fuel consumption, choose controls so that the control value divided by the product of the number of decisions remaining and the current velocity is held equal to a certain constant.
10.	10	To minimize fuel consumption, choose controls so that the current position divided by the product of the number of decisions remaining and the current velocity is held equal to a certain constant.

TABLE 1b. (Continued)

Number	Priority	Statement
11.	11	To minimize fuel consumption, choose controls so that the distance-to-go divided by the product of the number of decisions remaining and the current velocity is held equal to a certain constant.
12.	12	To minimize fuel consumption, choose controls so that the fuel consumption associated with each decision interval divided by the cumulated fuel consumption is held equal to a certain constant.
13.	13	To minimize fuel consumption, choose controls so that the product of the current acceleration and the number of decisions remaining divided by the current velocity is held equal to a certain constant.

the interpolation routine, etc. Because of the extensive space required, this type of analysis is not given for the remaining subtrajectories. For the subtrajectories, only the predicted heuristics and the total fuel consumption are reported and used in subsequent analyses.

Simplified Flow Chart

Figure 1 shows a simplified flow chart for the Mark I and Mark II models. The four modes of control appear in block A. The labels, A_1, $A_{2(a)}$, $A_{2(b)}$, and $A_{2(c)}$ are used in the flow chart to refer to the probing mode, terminal mode, heuristic mode, and gradient mode, respectively. The control value selected at each decision time is chosen in accord with one of these four control modes. After the parameters and initial conditions are set, the flow chart indicates the general procedure given below.

An initial control value of y_c is selected in accordance with the probing mode of control. As discussed in a later section, this initial value is always chosen to be equal to 0. The resulting updated values of the meter readings are then computed according to the equations shown in Figure 1 for the Mark I and Mark II models. Next, the flow chart shows that increments in the meter readings resulting from this control value are computed. These values are stored in the computer memory for later use. An extrapolation routine is then used to determine whether the final conditions of the problem can be met within the time available for the control problem. This extrapolation is not made, however, until every control has been used at least once in accord with the probing mode of control which is used in the initial stages of the control process. The feasibility of this procedure is assured under the assumption that the control problems are of long duration.

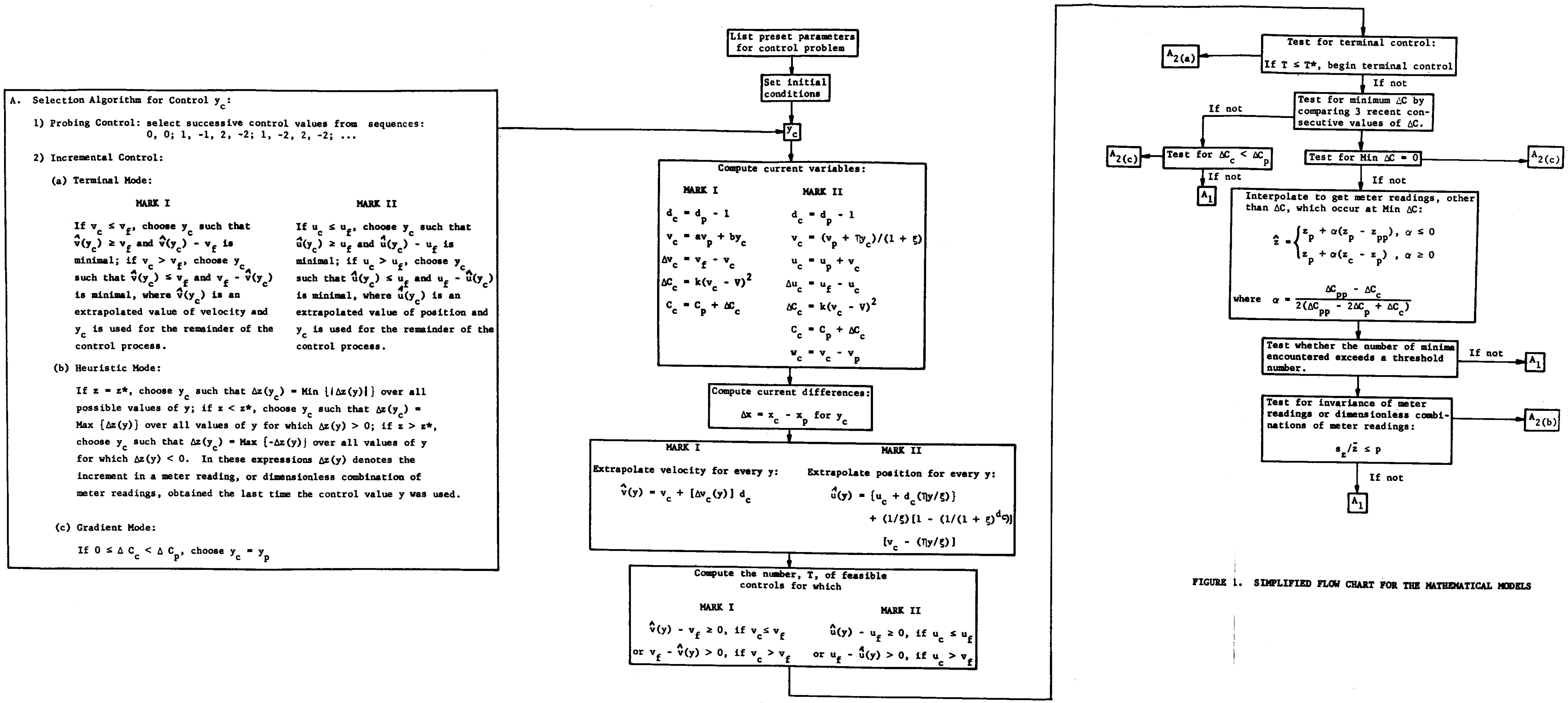


FIGURE 1. SIMPLIFIED FLOW CHART FOR THE MATHEMATICAL MODELS

After the extrapolations are made, the computer logic then determines whether the terminal mode of control should begin. This test is described in more detail below. If terminal control is required by the test, then the next contol value is selected in accord with the terminal control instructions contained in $A_{2(a)}$. If terminal control is not required, then the computer is instructed to test whether the cost increments have shown a minimum. This is carried out by examining the three most recent values of the cost increment. If no minimum is indicated, and the current cost increment ΔC_c is smaller than the previous cost increment ΔC_p, then the next control value is selected in accord with the instructions for gradient control contained in $A_{2(c)}$. If no minimum is indicated, and ΔC_c is not less than ΔC_p, then the next control value is selected in accord with the probing mode of control according to instructions contained in A_1. If a minimum is indicated and this minimum is zero, then a special version of the gradient control mode is used for this case. If a non-zero minimum occurs, then a simple quadratic interpolation routine is used to determine an estimated time at which the minimum occurred. The meter readings, other than the cost meter, are then interpolated linearly to obtain estimated values of these readings that existed when the cost increment was minimal. These interpolated values for each meter reading and dimensionless combination of meter readings are stored as a row in a matrix. When the number of rows in the matrix exceeds a threshold number, the numerical entries in the columns are examined for invariance. The test for invariance is made by computing the coefficient of variation $s_z/\bar{z}$ for each column of the matrix. If the coefficient of variation is less than, or equal to, a threshold p, then the next control value is selected in accord with the heuristic mode of control given in $A_{2(b)}$; otherwise, the probing mode of control given by A_1 is used.

This discussion of the structure of the flow chart of Figure 1 omits refinements which are exhibited in the complete computer program given in Appendix A. Although these refinements are necessary for the computer logic, the essential structure is that given in Figure 1. It is seen that the central theme of the computer program consists of obtaining a procedure which will generate a trajectory and, at the same time, will search for invariants, and use the invariants thereby found, if any, as a basis for the selection of controls.

In the Mark I model, the velocity is generally extrapolated before each control value is selected. The extrapolation is linear and is given by the following expression:

$$\hat{v}(y) = v_c + [\Delta v_c(y)]d_c \quad .$$

In this expression $\hat{v}(y)$ denotes the extrapolated velocity obtained when the control value y is used for the number of decision intervals d_c remaining in the control problem. The factor $\Delta v_c(y)$ is stored in the computer memory and denotes the change in the velocity that was obtained the last time the control value y was selected. These linear extrapolations are computed for each of the possible control choices to obtain $\hat{v}(-2)$, $\hat{v}(-1)$, $\hat{v}(0)$, $\hat{v}(1)$, and $\hat{v}(2)$.

In the Mark II model, the position variable is extrapolated. By means of the recursive relations given earlier (pages 28-32), the following extrapolation formula may be derived:

$$\hat{u}(y) = \{u_c + d_c(\eta y/\xi)\} + (1/\xi)[1 - (1 + \xi)^{d_c}][v_c - (\eta y/\xi)] \quad .$$

This exact extrapolation formula was used instead of an approximate linear extrapolation because of the expected difficulty of the Mark II control problems. For large values of the time constant it would be expected that the desired final position would be difficult to attain with the use of approximate extrapolation procedures. The next sections consist of a more detailed discussion of the algorithms for the four modes of control, and other computer programming procedures.

Algorithm for Probing Control

The algorithm for probing control serves as a basis for trial-and-error searching. This type of control is used at the beginning of each subtrajectory, and is used within a subtrajectory whenever none of the three remaining control algorithms is in use. In the probing control algorithm successive control values are selected from the sequence:

$$0,\ 0;\ 1,\ -1,\ 2,\ -2;\ 1,\ -1,\ 2,\ -2;\ .\ .\ .$$

At the beginning of the first subproblem this sequence is used until every possible control choice has been used. This procedure yields the control values 0, 0; 1, -1, 2, -2 for the first six selections without regard to fuel consumption. This procuedure is used in order that some data are obtained on each available control before a switch to another type of control is permitted.

Algorithm for Terminal Control

For the terminal mode in the Mark I model, the control value y_c is chosen to satisfy the following criteria:

$$\hat{v}(y_c) - v_f = \text{Min}\ \{\hat{v}(y) - v_f \mid \hat{v}(y) \geq v_f\}\ ,\ \text{if}\ v_c \leq v_f \quad ,$$

or

$$v_f - \hat{v}(y_c) = \text{Min}\ \{v_f - \hat{v}(y) \mid v_f \geq \hat{v}(y)\}\ ,\ \text{if}\ v_c > v_f \quad ,$$

where $\hat{v}(y)$ is the linearly extrapolated value of velocity which would be expected if y were used for the remainder of the control problem. In equivalent terms, that control is selected which will yield the final velocity early in time but early by the smallest amount. If there are no values of y which yield the final velocity within the desired time, then y_c satisfies the following criteria:

$$\hat{v}(y_c) - v_f = \text{Min}\ \{v_f - \hat{v}(y) \mid \hat{v}(y) < v_f\}\ ,\ \text{if}\ v_c \leq v_f \quad ,$$

or

$$\hat{v}(y) - v_f = \text{Min}\ \{\hat{v}(y) - v_f \mid \hat{v}(y) > v_f\}\ ,\ \text{if}\ v_c > v_f \quad .$$

For the terminal mode in the Mark II model, the control value y_c satisfies the same criteria as for Mark I except that positions replace velocities. That is, $\hat{v}(y)$, $\hat{v}(y_c)$, v_c, and v_f are replaced by $\hat{u}(y)$, $\hat{u}(y_c)$, u_c, and u_f, respectively.

As described above, it is necessary to determine whether or not the terminal mode of control should begin at each decision time of the control process. The test for initiating terminal control is performed as follows. An extrapolation is made to determine how many of the controls, if used repeatedly for the remainder of the control process, would attain the final conditions before the expiration of the remaining time. Let T denote the number of such controls.

Then terminal control is not initiated as long as T exceeds a threshold value, T*. If $T \leq T^*$, terminal control begins and the simulation models remain in this mode of control as long as T continues to be less than, or equal to, T*. However, at a subsequent decision time, if it is found that T again exceeds T*, then the process returns to its former mode of control (heuristic or probing). Thus, the terminal mode of control can be initiated and terminated several times during the generation of a trajectory. An exception to the above procedure occurs when the number of remaining decisions are less than, or equal to, a threshold value F. Then terminal control must be initiated, if not already initiated, and terminal control cannot again be terminated before the end of the trajectory.

The choice of a value for T* is partly determined by the number of controls available. In the Mark I and Mark II models only five control choices are available, and a value of 1 was taken to be a reasonable choice of T*. Thus, in the simulation models, if two or more controls yield extrapolated values reaching the desired levels within the remaining time, then the terminal mode of control does not begin. If the number of controls that yield acceptable extrapolations is less than, or equal to 1, then the terminal mode is initiated. The choice of a value for F is rather arbitrary. For sufficiently large values of F it can be insured that when terminal control is once initiated, it cannot be suspended at a later time in the trajectory. For smaller values of F, the suspension of terminal control may occur repeatedly. The value of F may be selected to yield the desired type of performance. For the Mark I and Mark II models the values of F were taken to be 3 and 6, respectively. These choices were made to allow some suspension of terminal control. Because of the increased difficulty of the Mark II problems, it seemed desirable to use a larger value of F for Mark II than that used for Mark I.

Algorithm for Heuristic Control

Suppose that a single meter, or combination of meters, is found to be a constant, z^*, when the incremental fuel costs are minimal. If the current value of this meter, or combination of meters is z, then two cases arise depending on whether $z = z^*$ or $z \neq z^*$. Suppose that the current value of z is equal to z^*. Then the control value is selected so that z changes as little as possible. To determine which control value to use, the computer program examines each stored increment in the z-value, $\Delta z(y)$, obtained the last time the control value y was used. The control is then determined as the value y_c for which

$$\Delta z(y_c) = \underset{\{y\}}{\mathrm{Min}} \{|\Delta z(y)|\} \quad .$$

If $z < z^*$, and $\Delta z(y) > 0$ for a given y, then such a choice of y would be expected to increase the value of z and may make z more nearly equal to z^*. That is, if y produced a positive increment in z the last time it was used, it would be expected to do so again. In the simulation models, all control values, y, for which $\Delta x(y) > 0$, are examined and that control which maximizes $\Delta z(y)$ is chosen; that is

$$\Delta z(y_c) = \underset{\{y\}}{\mathrm{Max}} \{\Delta z(y) \mid \Delta z(y) > 0\} \quad .$$

Similarly, if $z > z^*$, then y_c is chosen so that

$$\Delta z(y_c) = \underset{\{y\}}{\mathrm{Max}} \{-\Delta z(y) \mid \Delta z(y) < 0\} \quad .$$

It should be noted that this selection procedure is quite crude, any may yield overshoots, undershoots, and oscillations, particularly when z is nearly equal to z^*. To alleviate this difficulty, it is not required that z be be exactly equal to z^* in order to claim that the desired invariant relation is satisfied. Instead, z is regarded as equal to z^* in case

$$\left| z - z^* \right| < \epsilon \quad ,$$

where ϵ is a parameter of the models and may be set arbitrarily.

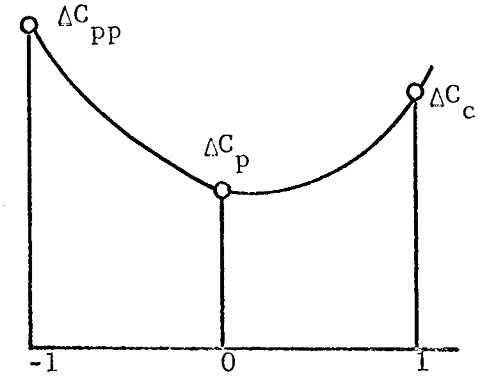
Algorithm for Gradient Control

In both simulation models, if the fuel cost associated with current decision interval is smaller than that of the preceding interval, then the last control choice is selected again. That is, as long as the incremental costs are decreasing, the same control choice is made. Because the fuel consumption is represented by a quadratic functional, the gradient mode of control can be expressed as follows. Choose $y_c = y_p$ whenever $0 \leq \Delta C_c < \Delta C_p$.

It may be noted that this type of control is not expected to yield optimal trajectories. It is incorporated into the simulation model in order to aid in the location of minima.

Interpolation Procedure

An interpolation procedure for the Mark I and Mark II models is used to obtain estimates of the meter readings which occurred whenever the process passed through a point having a minimum incremental fuel cost, ΔC. The interpolation involves fitting a parabola to three recent consecutive values of ΔC when the same control was used for the last two intervals. These three values constitute the current (ΔC_c), previous (ΔC_p), and pre-previous (ΔC_{pp}), values of the incremental fuel cost. The following sketch shows a normalized representation of such a minimum



The intervals, (-1,0) and (0,1), represent the previous and current decision intervals, respectively. The lengths of these intervals are constant and equal to the time between decisions.

To fit a parabola to these points, we use the form

$$Y = ax^2 + bx + c$$

and evaluate this form at the three points: $(-1, \Delta C_{pp})$, $(0, \Delta C_p)$ and $(1, \Delta C_c)$. This yields three simultaneous linear equations in the three unknowns, a, b, and c:

$$a - b + c = \Delta C_{pp}$$

$$c = \Delta C_p$$

$$a + b + c = \Delta C_c \quad .$$

The solution to these equations is given by

$$a = (\Delta C_{pp} - 2\Delta C_p + \Delta C_c)/2$$

$$b = (\Delta C_c - \Delta C_{pp})/2$$

$$c = \Delta C_p \quad .$$

Differentiation shows that the minimum occurs at

$$\alpha = -b/(2a) \quad ,$$

and substitution of the preceding values of a and b yields

$$\alpha = (\Delta C_{pp} - \Delta C_c)/2(\Delta C_{pp} - 2\Delta C_p + \Delta C_c)$$

provided

$$\Delta C_{pp} - 2\Delta C_p + \Delta C_c \neq 0 \quad .$$

Because the same control value has been used in these two decision intervals, it follows from the gradient mode of control that $\Delta C_p < \Delta C_{pp}$; otherwise, the control value would have been changed. Moreover, it follows that $\Delta C_p \leq \Delta C_c$ because the use of the same control was continued until the current cost increment exceeded, or equalled the previous cost increment. The addition of these two inequalities yields $2\Delta C_p < \Delta C_{pp} + \Delta C_c$, so that the denominator of the expression for α is positive, and hence, non-zero.

It is seen that the value of α is positive or negative depending on whether $\Delta C_{pp} > C_c$ or $\Delta C_{pp} < \Delta C_c$. That is, α is positive or negative depending on whether the minimum lies in the interval (0,1) or (-1,0). Moreover, the value of α indicates the "normalized time" at which the minimum occurred. For example, if $\alpha = 1/3$, then the minimum occurred at the time of the previous decision plus 1/3 of the time interval between successive decisions.

With the above value of α, the meter readings, other than ΔC, are interpolated linearly to obtain estimates of the meter readings which occurred at the time when the incremental fuel costs were minimal. This gives the following formulas for the linearly interpolated meter readings:

$$z = \begin{cases} z_p + \alpha(z_p - z_{pp}) & , \text{ if } \alpha \le 0 \\ z_p + \alpha(z_c - z_p) & , \text{ if } \alpha \le 0 \end{cases} ,$$

In these expressions z denotes a single meter reading or a dimensionless combination of meter readings. As each minimum occurs, the associated set of interpolated z-values are stored as a row of a matrix.

Finally, it should be noted that the above interpolation procedure is considerably modified in the event that the current incremental fuel cost is found to be equal to zero. In such a case, the meter readings at that time are simply scored as though they were obtained by the interpolation algorithm. It is convenient to count this procedure as an interpolation so that the number of interpolations is equal to the number of minima encountered during a control problem. As discussed in the next section, the examination for invariance begins when the number of interpolations has exceeded a threshold number.

Test for Invariance

As noted above, the interpolated values of the meter readings and dimensionless combinations of the meter readings are stored. For the Mark I model these stored values are not analyzed for invariance for the first time until seven interpolations have been carried out. In the Mark II model this number is reduced to 1. These threshold numbers are parameters of the simulation models and in the computer program are referred to as the "current memory limit", and are symbolized by CMLIM. In the Mark I model the value 7 was used as an estimated upper bound for the number of meter readings that a human controller might remember. The lower bound is clearly 1 since at least one minimum must be encountered before the meter readings can be associated with the occurrence of minimum fuel costs. As described below, the value of CMLIM can be reduced during a sequence of problems if invariant properties are found to exist.

The invariance sought among the stored meter readings is of the simplest type. It is asked whether any of the meters shows a constant value when the incremental fuel cost is a minimum. If not, it is next determined whether any of the dimensionless combinations of meter readings is constant when the incremental fuel cost is a minimum. If no such invariance is found, the simulation models return to the probing mode of control. If an invariance is found, then the simulation models select controls in accord with the heuristic control algorithm.

The stored meter values associated with the first occurrence of a minimum are represented by the values in the first row of a matrix. The corresponding meter values associated with the occurrance of the second minimum are stored in the second row, etc. The test for invariance then consists of an examination of the numerical values in the columns of the matrix of stored values. Because these stored numbers are estimated by the interpolation algorithm, it is not expected that strict constancy would be found using this approach, even if such constancy were theoretically correct. To permit some degree of variability we assume that if the standard deviation

$$s_{z_j} = (1/(n-1)) \left(\sum_{i=1}^{m} (z_{ij} - \bar{z}_j)^2 \right)^{1/2} \quad , \quad j = 1,\ldots,m$$

is sufficiently small, then the readings z_j, on the j^{th} meter are constant over the m minima encountered. To further normalize over the meters it is convenient to use the coefficient of variation given by

$$\delta_j = s_{z_j}/\bar{z}_j \quad .$$

The tests for invariance are carried out by assigning a threshold value, p, to the coefficient of variation. If $\delta_j \le p$, then the j^{th} meter, or combination of meters, is assumed to be constant with a value equal to $\bar{z}_j$ when the incremental fuel costs are minimal.

Suppose that a particular meter, or combination of meters, shows an invariance for two successive control problems. If, in addition, the successive mean values are nearly equal, then the invariance is said to be "strong"; if the meter readings have small coefficients of variation, but different mean values for the two problems, then the invariance is said to be "weak". In other words, strong invariance occurs when the relevant meter readings are constant within and between successive trajectories; weak invariance occurs when the relevant meter readings are constant within, but not between, successive trajectories. In either case the existence of invariance assures that the heuristic mode of control will be used. If the use of the heuristic mode of control does not result in increasing incremental fuel costs, then the heuristic mode is successful. The computer logic is structured so that successful heuristic control yields a reduction in the number of interpolations required in the next problem before a transition is permitted from probing control to heuristic control. In the simulation models, if the invariance found in two successive trajectories is strong, then the current memory limit for the next problem is given by

$$\mathrm{Max}\{\mathrm{CMLIM} - 2,\ 1\} \quad .$$

Thus, a strong invariance reduces the memory limit by 2, but never below 1. Similarly, if the invariance is found to be weak, then the memory limit for the next subtrajectory is given by

$$\mathrm{Max}\{\mathrm{CMLIM} -1, 1\} \quad .$$

If no invariance is found, no change is made in the memory limit. This change in the memory limit is the only "adaptive" characteristic of the logic used in the Mark I and Mark II models.

Definitions of Input Parameters

Table 2 shows a detailed listing of the preset parameters for the control problems. As shown by the flow chart of Figure 1, these parameters must be set at the beginning of the computations. The symbols which appear in column 3 correspond to those used in the computer instructions. The numerical values of these symbols completely define the 23 Mark I problems and the 12 Mark II problems simulated by the computer.

The table shows that the number of meters M is equal to 23 and 12. The memory limit CMLIM is 7 and 1 for the two models. The value of F is represented by FINTERM and is seen to be equal to 3 and 6 as discussed in the section concerned with the test for initiating terminal control. The value of T is equal to 1 for both models and is denoted by TCRT in Table 2. The dimensional analysis procedure requires the rank of the dimensional matrix, the order of the P-matrix, and the number of rows and columns of the Q-matrix. These are given by R, IPSIZE, IQSIZE, and JQSIZE in the table.

TABLE 2 . INPUT PARAMETERS

Number	Parameter Definition	Symbol	Numerical Value Mark I	Numerical Value Mark II
1	Number of Meters	M	6	8
2	Number of Subtrajectories	PMAX	23	12
3	Initial Number of Interpolations Required to Begin Analysis	CMLIM	7	1
4	Number of Decisions Remaining when Terminal Control Must Begin	FINTERM	3	6
5	Threshold Number of Controls	TCRT	1	1
6	Rank of Dimensional Matrix	R	3	3
7	Order of P-Matrix	IPSIZE	3	3
8	Number of Rows of Q-Matrix	IQSIZE	3	5
9	Number of Columns of Q-Matrix	JQSIZE	3	3
10	Initial Value of Variable to be Controlled:			
	Subtrajectory 1	GOAL(1)	470.	1100.
	" 2	" (2)	590.	1148.
	" 3	" (3)	470.	1100.
	" 4	" (4)	580.	1260.
	" 5	" (5)	480.	960.
	" 6	" (6)	570.	260.
	" 7	" (7)	490.	560.
	" 8	" (8)	560.	640.
	" 9	" (9)	710.	660.
	" 10	" (10)	630.	460.
	" 11	" (11)	830.	320.
	" 12	" (12)	610.	410.
	" 13	" (13)	650.	--
	" 14	" (14)	630.	--
	" 15	" (15)	670.	--
	" 16	" (16)	650.	--
	" 17	" (17)	670.	--
	" 18	" (18)	670.	--
	" 19	" (19)	610.	--
	" 20	" (20)	650.	--
	" 21	" (21)	670.	--
	" 22	" (22)	620.	--
	" 23	" (23)	650.	--
11	Final Value of Variables to be Controlled:			
	Subtrajectory 12	" (13)	--	825.
	" 23	" (24)	570.	--

TABLE 2 . (Continued)

Number	Parameter Definition	Symbol	Numerical Value Mark I	Mark II
12	Number of Decisions			
	Subtrajectory 1	DEC (1)	38	20
	" 2	" (2)	34	20
	" 3	" (3)	30	20
	" 4	" (4)	26	20
	" 5	" (5)	22	20
	" 6	" (6)	18	20
	" 7	" (7)	14	20
	" 8	" (8)	18	20
	" 9	" (9)	17	20
	" 10	" (10)	16	20
	" 11	" (11)	16	20
	" 12	" (12)	16	20
	" 13	" (13)	15	--
	" 14	" (14)	14	--
	" 15	" (15)	13	--
	" 16	" (16)	12	--
	" 17	" (17)	11	--
	" 18	" (18)	10	--
	" 19	" (19)	9	--
	" 20	" (20)	9	--
	" 21	" (21)	9	--
	" 22	" (22)	8	--
	" 23	" (23)	8	--
13	Parameters of Transformation Laws:			
	$\underline{P}$ = 1, 2, ..., 23	LCA($\underline{P}$)	1.0	--
	$\underline{P}$ = 1, 2, ..., 23	LCB($\underline{P}$)	10.0	--
	$\underline{P}$ = 1, 2, 8, 11	CSI($\underline{P}$)	--	0.4140
	$\underline{P}$ = 3, 4, 7, 10	CSI($\underline{P}$)	--	0.0908
	$\underline{P}$ = 5, 6, 9, 12	CSI($\underline{P}$)	--	0.0444
	$\underline{P}$ - 1, 2, ..., 12	ETA($\underline{P}$)	--	1.0
14	Cost Increment Factor			
	Subtrajectory 1	V(1)	1.0	1.0
	" 2	V(2)	1.0	1.0
	" 3	V(3)	0.1	1.0
	" 4	V(4)	5.0	1.0
	" 5	V(5)	1.0	1.0
	" 6	V(6)	0.5	1.0
	" 7	V(7)	1.0	1.0
	" 8	V(8)	1.0	1.0
	" 9	V(9)	0.1	1.0
	" 10	V(10)	0.5	1.0
	" 11	V(11)	1.0	1.0

TABLE 2. (Continued)

Number	Parameter Definition	Symbol	Numerical Value Mark I	Numerical Value Mark II
	Cost Increment Factor (Continued)			
	Subtrajectory 12	V(12)	0.5	1.0
	" 13	V(13)	1.0	--
	" 14	V(14)	5.0	--
	" 15	V(15)	0.3	--
	" 16	V(16)	2.0	--
	" 17	V(17)	1.2	--
	" 18	V(18)	2.0	--
	" 19	V(19)	0.2	--
	" 20	V(20)	4.0	--
	" 21	V(21)	2.0	--
	" 22	V(22)	0.6	--
	" 23	V(23)	3.0	--
15	Miss-distance Cost Factor P = 1, 2, ..., P MAX	W(P)	100	100
16	Reference Levels for Variables to be Controlled:			
	Subtrajectory 1	A(1)	570.0	2.4
	" 2	A(2)	500.0	-2.4
	" 3	A(3)	550.0	8.0
	" 4	A(4)	520.0	-15.0
	" 5	A(5)	500.0	-35.0
	" 6	A(6)	558.0	15.0
	" 7	A(7)	513.0	4.0
	" 8	A(8)	740.0	-2.0
	" 9	A(9)	580.0	-10.0
	" 10	A(10)	730.0	-7.0
	" 11	A(11)	780.0	3.5
	" 12	A(12)	730.0	25.0
	" 13	A(13)	540.0	--
	" 14	A(14)	610.0	--
	" 15	A(15)	690.0	--
	" 16	A(16)	700.0	--
	" 17	A(17)	630.0	--
	" 18	A(18)	680.0	--
	" 19	A(19)	600.0	--
	" 20	A(20)	620.0	--
	" 21	A(21)	685.0	--
	" 22	A(22)	598.0	--
	" 23	A(23)	650.0	--
17	Parameter Determining the Number of Controls, P = 1, 2, ..., P MAX	C(P)	2	2
18	Threshold Coefficient of Variation, P = 1, 2, ..., P MAX	PSTAR	0.010	0.025
19	Lower Cost-Free Error Limit for Endpoint of Subtrajectory, P = 1, 2, ..., P MAX	LOWL(P)	0.00	5.0

TABLE 2 . (Continued)

Number	Parameter Definition	Symbol	Numerical Value Mark I	Numerical Value Mark II
20	Upper Cost-Free Error Limit for Endpoint of Subtrajectory, P = 1, 2, ..., P MAX	UPL(P)	0.00	5.0
21	Initial Velocity			
	Subtrajectory 1	VEL(1)	--	0.0
	" 2	" (2)	--	0.0
	" 3	" (3)	--	5.0
	" 4	" (4)	--	-10.0
	" 5	" (5)	--	-40.0
	" 6	" (6)	--	20.0
	" 7	" (7)	--	-4.0
	" 8	" (8)	--	1.0
	" 9	" (9)	--	0.0
	" 10	"(10)	--	-1.0
	" 11	"(11)	--	2.0
	" 12	"(12)	--	20.0
22	Initial Acceleration, P = 1, ..., 12	ACCEL(P)	--	0.0
23	Cost-Free Error Allowed for Invariants, P = 1, ..., P MAX	EPI(P)	0.025	0.025

The parameter to be controlled is velocity in the Mark I model and position in the Mark II model. The initial values for the controlled variables for each subtrajectory are given by GOAL(P), where P = 1,...,23 for Mark I and P = 1,...,12 for Mark II. In general, the final value for a controlled variable is equal to the initial value of the next subtrajectory. The number of decisions for each subtrajectory is represented by DEC(P), with P = 1,...,23 for Mark I and P = 1,...,12 for Mark II. The table shows that the number of decisions varies between 8 and 38 for Mark I and is equal to 20 for every subtrajectory of Mark II.

The values of $\underline{a}$ and $\underline{b}$ of the Mark I transformation law, $v_{k+1} = av_k + by_{k+1}$, are denoted by LCA(P) and LCB(P), respectively, for P = 1,...,23. Table 2 shows that these values are equal to 1.0 and 10.0 for all Mark I problems. The values of the three reciprocal time constants ξ are given by CSI(P). The table shows that CSI(p) = 0.4140 for P = 1,2,8,11; CSI(p) = 0.0908 for P = 3,4,7,10; and CSI(P) = 0.0444 for P = 5,6,9, and 12. The value of η is represented by ETA(P) and is seen to be equal to 1.0 for all 12 Mark II trajectories.

The values of A and B in the Mark I objective functional,

$$C = A \sum_{k=1}^{N} (v_k - V)^2 + B(v_N - v_f)^2$$

are represented by V(P) and W(P), respectively. The table shows that the values of V(P) vary between 0.1 and 5.0 and that W(P) = 100 for every Mark I subtrajectory. For the Mark II subtrajectories V(P) = 1.0 and W(P) = 100 for P = 1,...,12. The reference velocity level V of the objective functional given above is given by A(P) for the 23 subtrajectories of the Mark I model and the 12 subtrajectories of the Mark II model.

Table 2 shows a parameter C(P) that determines the number of control values available for each problem. The computer logic was structured to permit control values contained in the following set:

$$S\{0, \pm 1, \pm 2, \ldots, \pm C(P)\}$$

The number of controls is then given by 2C(P) + 1. For the Mark I and Mark II models, C(P) is equal to 2, so the 5 control values are contained in the set:

$$S\{-2, -1, 0, 1, 2\}$$

The threshold value p of the coefficient of variation used in the tests for invariance is represented by PSTAR in Table 2. The value is seen to be equal to 0.010 for the Mark I problem and is equal to 0.025 for the Mark II problems.

In the Mark I problems, it is required that each terminal velocity be exactly achieved. Otherwise the miss-distance penalty, $B(v_N - v_f)^2$, is imposed on the fuel cost. This is shown in Table 2 by the fact that lower and upper limits, LOWL(P) and UPL(P), are equal to zero for every problem. For the Mark II problems, however, these errors must exceed 5.0 before the miss-distance penalty is imposed.

The Mark II problems require the specifications of initial velocities and initial accelerations. These are given by VEL(P) and ACCEL(P) in Table 2.

The last entry in Table 2 shows that the ε-error allowed in satisfying an invariant relation is represented by EPI(P) and is equal to 0.025 for both the Mark I and Mark II problems.

An Example of Simulation Results

An example of the computer simulations is given below for control problem number 10 of the Mark I set of problems.

Table 3 shows the information printed out at the beginning of each Mark I control problem. The values shown in this list are obtained from those given in Table 2.

Table 4 shows a sample of output for the six meter readings resulting from each control value selected by the Mark I simulation for stubtrajectory number 10. The control value selected is shown by meter 1 and the resulting up-dated values are shown on meters 2 through 6. This table does not show the control mode that serves as a basis for the selection of each control value. However, a detailed computer print-out permits this information to be extracted.

Table 5 shows the control mode associated with each control value for the Mark I simulation for subtrajectory 10. The selection of y = 1 for the first control value is required for every subproblem except the first, where the control value is zero. This first control value is obtained from the first element of the probing sequence: 1, -1, 2, -2. Since probing control is not used again in this subtrajectory, the second element, y = -1, is not slected from this sequence. Before the selection of the next control value, the simulation model determines that only one control, y = 2, used repeatedly, can yield the desired final velocity within the remaining number of decisions. Thus, T = 1 and since the threshold value of T given in Table 2 as TCRT, is also equal to 1, terminal control begins. This mode of control continues through decision 5. For decision 6, however, it is found that T = 2, so that terminal control is suspended. Moreover, since the last choice, y = 2, resulted in a decrease of the incremental fuel cost from 450 to 50, the gradient mode of control choice also yields an

TABLE 3 . SAMPLE OUTPUT OF PARAMETER INFORMATION FOR SUBTRAJECTORY NUMBER 10

INITIAL VALUE OF STATE VARIABLE, GOAL(10)	630.00
DESIRED FINAL VALUE OF STATE VARIABLE, GOAL(11)	830.00
DECISIONS AVAILABLE TO REACH FINAL VALUE, DEC(10)	16
REFERENCE LEVEL, A(10)	730.00
MEMORY LIMIT, CMLIM	1
THRESHOLD COEFFICIENT OF VARIATION, PSTAR	0.010
NUMBER OF CONTROLS TO INITIATE TERMINAL CONTROL, TCRT	1
LOWER LIMIT ON FINAL VALUE OF STATE VARIABLE, LOWL(10)	0.00
UPPER LIMIT ON FINAL VALUE OF STATE VARIABLE, UPL(10)	0.00
COEFFICIENT OF PREVIOUS STATE VALUE, LCA(10)	1.0
COEFFICIENT OF CONTROL VALUE, LCB(10)	10.0
COEFFICIENT OF COST INCREMENTS, V(10)	0.5
COEFFICIENT OF FINAL MISS-DISTANCE, W(10)	100.

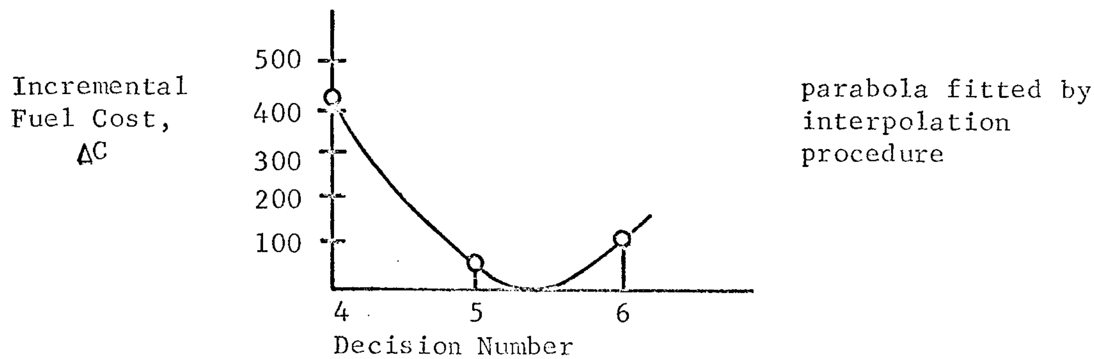
TABLE 4. SAMPLE OUTPUT FOR THE SIX METER READINGS RESULTING FROM EACH CONTROL VALUE SELECTED BY THE MARK I SIMULATION FOR SUBTRAJECTORY NUMBER 10

Control Choice (Meter 1)	Remaining Decisions (Meter 2)	Current Velocity (Meter 3)	Velocity Increment To Go (Meter 4)	Incremental Fuel Cost (Meter 5)	Cumulative Fuel Cost (Meter 6)
1	15	640	190	4050	4050
2	14	660	170	2450	6500
2	13	680	150	1250	7750
2	12	700	130	450	8200
2	11	720	110	50	8250
2	10	740	90	50	8300
0	9	740	90	50	8350
0	8	740	90	50	8400
2	7	760	70	450	8850
-2	6	740	90	50	8900
2	5	760	70	450	9350
2	4	780	50	1250	10600
2	3	800	30	2450	13050
1	2	810	20	3200	16250
1	1	820	10	4050	20300
1	0	830	0	500	25300

TABLE 5 . CONTROL MODES USED BY THE MARK I SIMULATION FOR SUBTRAJECTORY NUMBER 10

Decision Number	Control Mode	Control Choice (Meter 1)	Remaining Decisions (Meter 2)
1	Probing	1	15
2	Terminal	2	14
3	Terminal	2	13
4	Terminal	2	12
5	Terminal	2	11
6	Gradient	2	10
7	Heuristic	0	9
8	Heuristic	0	8
9	Terminal	2	7
10	Heuristic	-2	6
11	Terminal	2	5
12	Terminal	2	4
13	Terminal	2	3
14	Terminal	1	2
15	Terminal	1	1
16	Terminal	1	0

incremental fuel cost of 50. This suggests that a minimum exists in the incremental cost function as shown by the following sketch.



A fitted parabola yields a minimum at decision number 5.5. Thus, the remaining meter readings are linearly interpolated to obtain estimates of their values when half of the time has elapsed between decision 5 and decision 6. From lines five and six of Table 4, it is seen that these interpolated values are 2, 10.5, 730, 100, and 8275 for meters 1, 2, 3, 4, and 6, respectively. These values are computed and stored. Now Table 3 shows that the memory limit is equal to 1 for this problem, so that only one minimum is required in order to initiate the heuristic mode of control. Because all five of these meters are (trivially) constant over the required number of minima, there are five possible heuristics corresponding to these five meters. Moreover, every dimensionless product formed from these values is constant so that every heuristic in the list of possible heuristics shown in Table 1 is not admissible. This table shows that the heuristic having highest priority is that based on meter 3, and for the present problem it takes the following form:

> To minimize fuel consumption, choose control values so that the current velocity, shown on meter 3, is made equal to 730.

Because the current velocity at this time is equal to 740 and this value differs from the desired velocity of 730 by less than 2.5 percent, the model assumes that the heuristic is satisfied. Thus, the control values are then searched to determine which control would be expected to produce the smallest change in the current velocity. The stored increments of velocity, $\Delta v(y)$, associated with the most recent use of each control y are examined. In this way the control choice $y_c = 0$ is found to be appropriate for decision 7.

Decision 8 is also made to keep the current velocity unchanged in accordance with heuristic number 3. Thus, decision 8 also yields $y_c = 0$. For decision 9 the model determines that terminal control must again begin and selects y = 2. For decision 10 terminal control is again suspended, and the heuristic mode of control yields $y_c = -2$. This choice is based on heuristic number 4, of second highest priority, associated with making meter 4 read 100. Finally, the last six decisions are made under the terminal mode of control because F, given in Table 2 as FINTERM, is equal to 6. The change from y = 2 to y = 1 at decision 13 results from the fact that both y = 1 and y = 2 would achieve the desired final velocity within the required number of decisions, but y = 1 yields a smaller early arrival time. In fact, y = 1 yields the desired final velocity at the required time with a miss-distance equal to zero.

Figure 2 shows the resulting plot of velocity as a function of the number of decisions remaining as obtained by the Mark I simulation for problem number 10.

Predicted Heuristics Obtained from Mark I Simulation

The heuristics obtained from these simulations serve as predictions of those that will be verbalized by human controllers. In particular, based

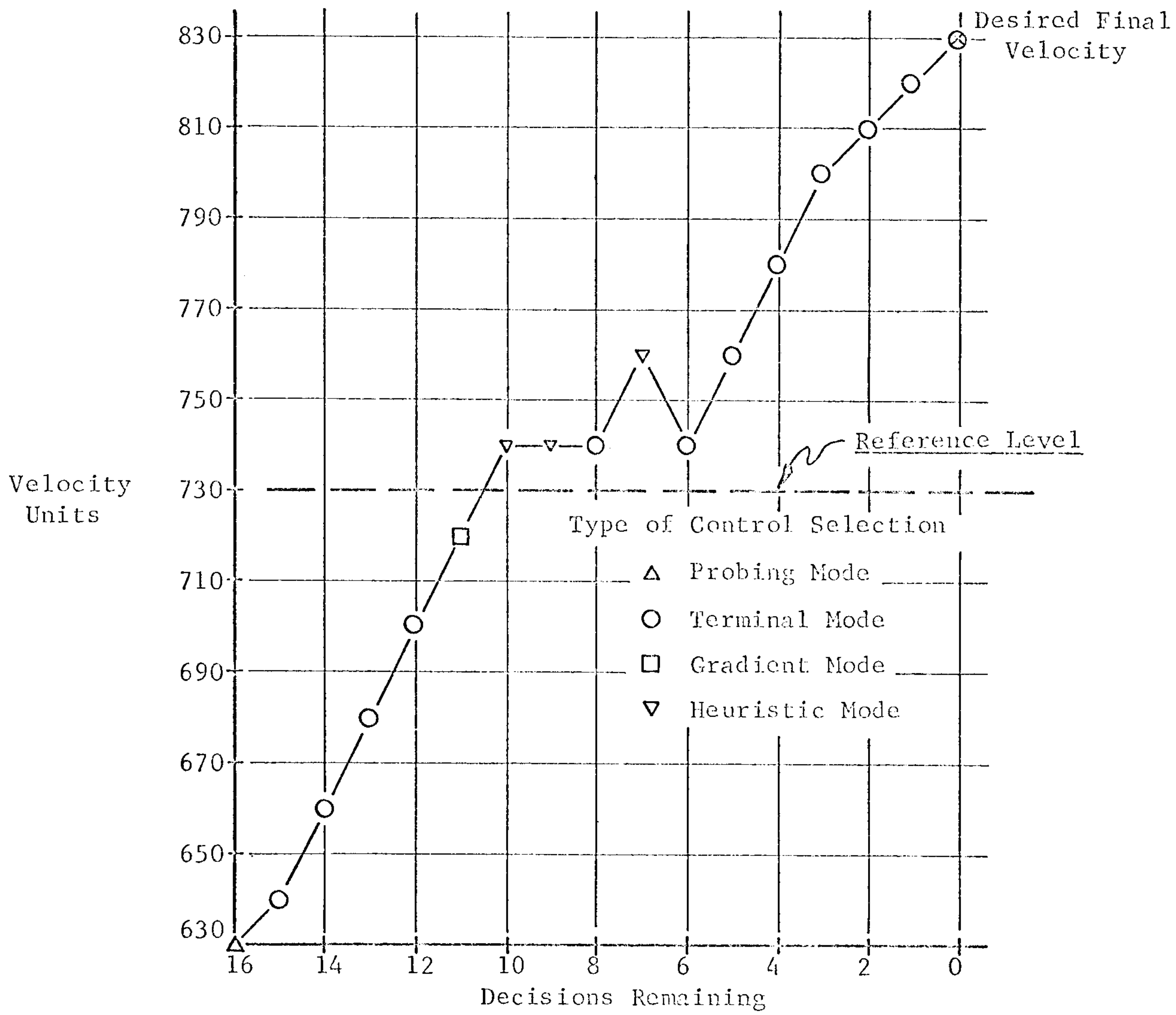


FIGURE 2. SUBTRAJECTORY GENERATED BY MARK I MODEL FOR PROBLEM NUMBER 10

on the simulation for subtrajectory 10, it is predicted that heuristics number 3 and number 4 will be verbalized by human controllers. By applying the simulation model to the 23 Mark I problems, a set of 23 predictions is obtained.

Table 6 shows a listing of the heuristics obtained by the Mark I simulation for the 23 subtrajectories. The numbers refer to those given in Table 1a. It is seen that no heuristic is obtained for subtrajectories numbered 9, 11, 13, 20, 21, and 22. This results from the use of the gradient mode of control with a control value that reduced incremental fuel costs as desired. However, the rate of reduction of fuel costs is so slow that final terminal control must begin before a minimum fuel increment is detected. As noted above for subtrajectory 10, subtrajectories 12 and 18 yield changes from heuristic three to a period of terminal control followed by another period of heuristic control based on heuristic four. With the current computer logic this results in the use of a second heuristic having the next lower priority.

EXPERIMENTAL STUDIES WITH HUMAN CONTROLLERS

The following sections present the procedures and some results of experimental studies with human controllers. In performing the experiments, we made the following test hypothesis: The subjects would discover the existence of the reference velocity, would verbalize their discovery as a recommendation for a control heuristic, and would use the heuristic to guide their own selection of controls. To obtain data on the proposed models for the prediction of verbal heuristics, the control problems listed in Table 2 were presented to 26 subjects. As previously discussed, these control problems are structures so that the incremental fuel "cost", $A(v_k - V)^2$, is minimized by maintaining the current velocity

TABLE 6 . HEURISTICS USED BY MARK I SIMULATIONS

Subtrajectory Number	Heuristic Number	Subtrajectory Number	Heuristic Number
1	3	13	none
2	3	14	3
3	3	15	3
4	3	16	3
5	3	17	3
6	3	18	3,4**
7	3	19	3
8	3	20	none*
9	none*	21	none*
10	3,4**	22	none*
11	none	23	3
12	3,4**		

* The model entered final terminal control before reaching a minimum incremental fuel consumption.

** The change from heuristic 3 to heuristic 4 resulted from an intervening initiation and suspension of terminal control.

v_k equal to the reference velocity V. This structure was not known to the subjects and was not utilized in the computer simulation logic. Fourteen subjects were employed for the Mark I experiments and for the Mark II experiments. The systems to be controlled are simulated on a CDC 3400 computer. The subjects made successive decisions based upon the simulated meter readings. The verbal statements made by the subjects were recorded for further analysis.

Description of the Experiment

Experimental studies were performed with 14 subjects as human controllers for the Mark I control problems and for the Mark II control problems. The subjects for the Mark II problems represented a higher level of training than the Mark I subjects. As shown in Figure 3 each subject was seated before a Control Data Typewriter console (3401). A set of instructions, given in Appendix C, was read to each subject and questions regarding the task were answered. The task was described as one in which the subject had to control a simulated space vehicle through a sequence of trajectories while minimizing total "fuel" consumption.

Figure 4 shows that the five control values available to the subject could be selected by depressing the appropriate keys on the typewriter console. Ten seconds after the meter readings were printed out, the subject had a five second interval in which he was instructed to make his next selection. This five second period was indicated by switching on a small light placed above the typewriter keyboard. If a subject failed to enter a decision before the end of this period, the experimenter entered the previous selection of the subject. This occurred 22 times out of a total of 8,722 selections during the experiments.

FIGURE 3. SUBJECT SEATED BEFORE TYPEWRITER CONSOLE

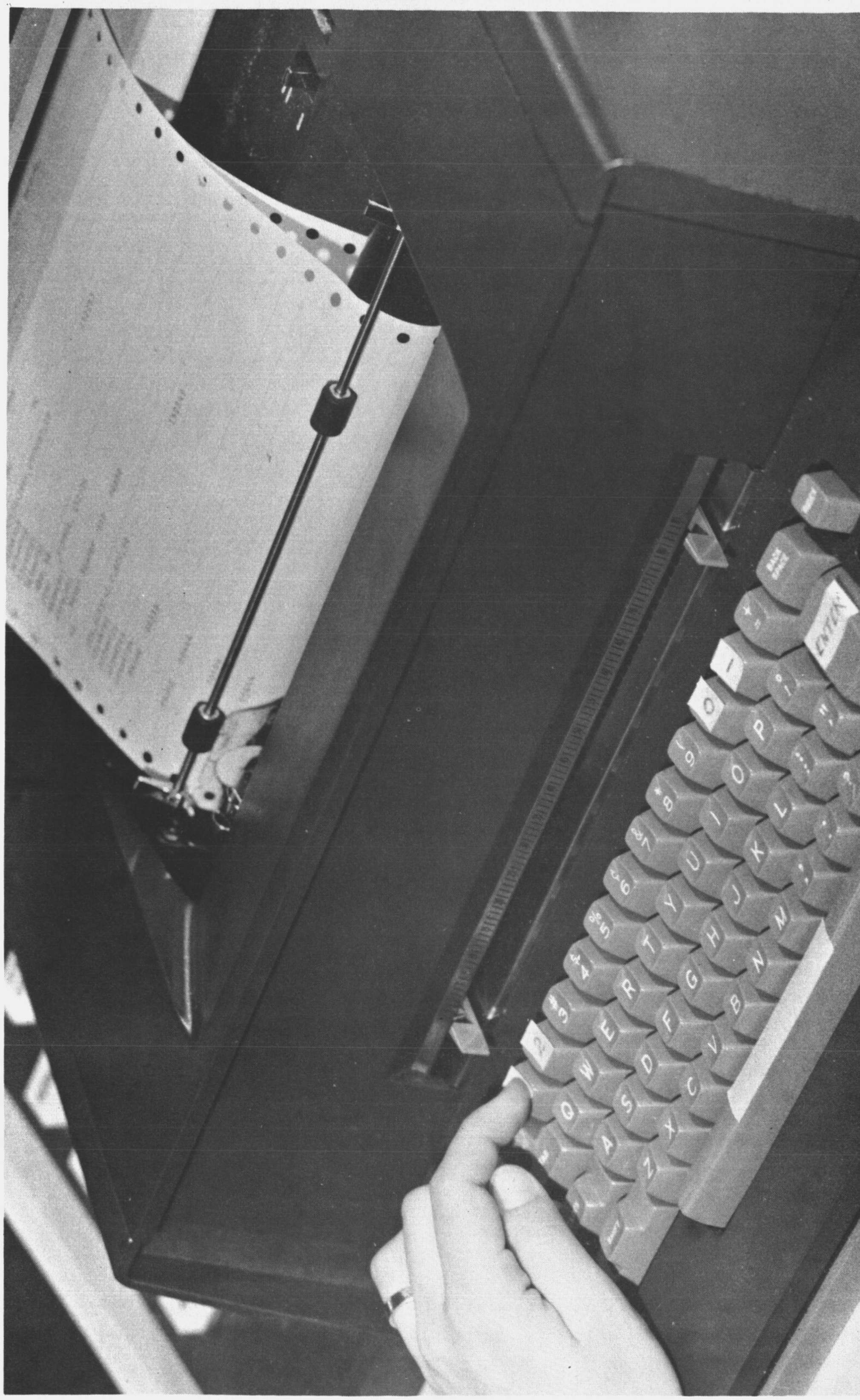


FIGURE 4. SUBJECT MAKING ENTRY ON TYPEWRITER KEYBOARD

The subjects for the Mark I experiments consisted of 14 students from The Ohio State University. They ranged from third quarter freshmen to first quarter graduate students and were enrolled in both science and non-science curricula. The subjects were obtained by placing an advertisement in the university newspaper and were paid $5.00 each for participating in the experiment.

The 14 subjects for the Mark II experiments generally represented a higher level of training than the Mark I subjects. Two of the subjects served in the Mark I experiment; two more held Ph.D. degrees in physics; two more were Ph.D. students in psychology. The remaining subjects were enrolled in a variety of fields. With the exception of the two Ph.D. physicists, each subject was paid $5.00 for participating in the experiment.

Approximately 1-1/2 hours were required by each subject to complete the 23 Mark I trajectories. For the 12 Mark II trajectories, the average time per subject was 45 minutes. After completing the experiment, each subject was asked not to discuss the project with anyone else who might be a subject.

After a control selection was made on the typewriter console, a Control Data 3400 computer tabulated the values of the current variables in accord with the transformation equations for the meter readings. Two copies of the updated meter readings were printed out. One of these copies appeared in the typewriter console and served as the record of progress of the subjects.

Figure 5 shows a typical print-out for a Mark I control problem. The subproblem number, initial velocity, desired final velocity, and the total number of decisions available are shown at the top of the page. The six columns at the left side of the page give the following "meter" readings: (1) number of

```
SUB-PROBLEM NO.  19
INITIAL VELOCITY -    610
FINAL VELOCITY -   650
NO. OF DECISIONS -    9

 NO. CUR. DIST. CV  COST   CUM.  440  490  540  590  640  690  740  790
DEC. VEL.  F.V.           COST
      610   -40        0      0                   X   O

  9              2

      630   -20      180    180                     X O

  8              1

      640   -10      320    500                      XO

  7             -2

      620   -30       80    580                    X  O

  6             -2

      600   -50        0    580                  X    O

  5              0

      600   -50        0    580                  X    O

  4              0

      600   -50        0    580                  X    O

  3              1

      610   -40       20    600                   X   O

  2              2

      630   -20      180    780                     X O

  1              2

      650     0      500   1280                       X

PENALTY =          0
FINAL CUMULATIVE COST =    1280
 PAUSE
        OK
```

FIGURE 5. EXAMPLE OF SUBJECTS' DATA SHEET FOR MARK I EXPERIMENTS

decisions remaining, (2) current velocity, (3) "distance" to go to reach final velocity, (4) control value selected, (5) fuel cost for last control selection, and (6) accumulated fuel cost. The right-hand portion of the print-out displays a graphical representation of the trajectory of the subject as it develops. The X's show the sequence of current velocities resulting from the control choices of the subjects. The O's show the level of the desired final velocity.

The miss-distance penalty, for failing to achieve the desired final velocity, is printed-out at the bottom of the page at the completion of the problem. Because the trajectory of the subject shown in Figure 5 achieved the desired final velocity, the penalty shown is zero. The cumulative cost shown in the print-out is the sum of all the incremental costs, shown in column 5, and the miss-distance penalty.

Figure 6 shows an example of a data sheet for a Mark II experiment. The information displayed as meter readings is similar to that of the Mark I experiments. The graphical display of the trajectory was omitted because of the space limitations of the typewriter console.

The verbal recommendations of the subjects were recorded for further analysis. Figure 7 shows a subject "radioing back" information which he believes would be of assistance to another astronaut about to begin a similar flight. At the end of each subtrajectory, the subjects were asked to relay such information and to state their confidence in it. These statements were tape recorded to permit subsequent analysis for the agreement between the recommendations of the subjects for control selection and the predicted heuristics yielded by the simulation models.

TRAJECTORY NO. 3

INITIAL POSITION 1100.00
FINAL POSITION 1260.00

INITIAL VELOCITY 5.00
INITIAL ACCELERATION 0.00
SYSTEM RESPONSE - MEDIUM (TIME CONSTANT - MEDIUM)

NO. OF DECISIONS 20

REMAINING DECISIONS	CONTROL CHOICE	CURRENT POSITION	CURRENT VELOCITY	CURRENT ACCEL.	DISTANCE TO GO	FUEL COST	CUMULATIVE FUEL COST
		1100.00	5.00	0.00	160.00	0.00	0.00
20	0						
		1104.58	4.58	-0.42	155.42	11.67	11.67
19	-2						

		1143.33	8.07	1.27	116.67	0.00	116.58
11	2						
		1152.56	9.23	1.16	107.44	1.51	118.09
10	1						
		1161.93	9.38	0.15	98.07	1.89	119.99
9	1						
		1171.45	9.51	0.14	88.55	2.29	122.27
8	1						
		1181.08	9.64	0.12	78.92	2.68	124.96
7	1						
		1190.84	9.75	0.11	69.16	3.07	128.03

2	2						
		1253.32	14.08	0.72	6.68	36.04	236.00
1	-2						
		1264.40	11.07	-3.01	-4.40	9.44	245.45

PENALTY = 0.00
FINAL CUMULATIVE COST = 245.45
PAUSE OK

FIGURE 6. EXAMPLE OF SUBJECTS' DATA SHEET FOR MARK II EXPERIMENTS

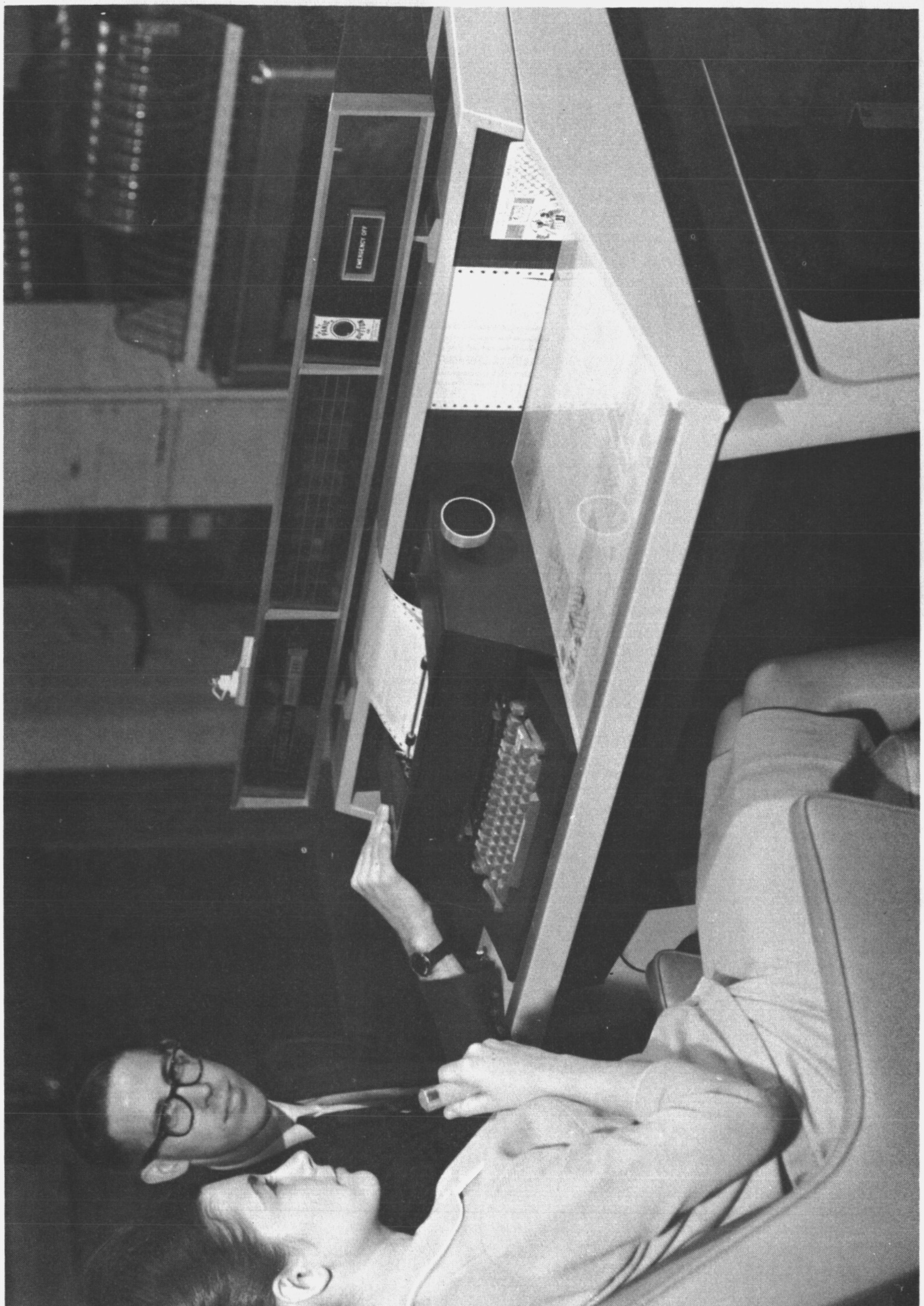


FIGURE 7. SUBJECT "RADIOING BACK" INFORMATION

Experimental Results

As a means of analyzing the data, plots were made of the trajectories of each subject. Shown in Figure 8 are the simple sketches of the 14 trajectories generated by the subjects for problem 10 of the Mark I series. The ideal trajectory that minimizes the total fuel cost and the trajectory generated by the Mark I simulation model are also given in the figure. It is seen that Subjects 1, 2, and 4 generated minimum cost trajectories, and the trajectories generated by Subjects 3, 6, and 13 are nearly minimum cost trajectories. A close similarity exists between the trajectory generated by Subject 12 and that generated by the simulation model.

Typewritten copies of the verbal statements of the subjects were made. The verbal statements made by the 14 subjects for problem 10 are listed in Table 7. In generεl, it can be seen that those subjects with minimum, or near minimum, costs used some form of the predicted heuristic generated by the simulation model as discussed in the simulation example (Pages 56-61). The heuristic of Subject 9, "Make the slope very slight", is well depicted by his trajectory. This subject used the same heuristic for all 23 problems.

Copies of the verbal statements were given to three people to classify. Two of these persons had had no previous connection with the project. The third had served as the experimenter. Each person was given a list of the possible model heuristics for Mark I (Table 1a). Their task was to read each verbal statement and decide whether or not it was equivalent to any of those on the list. If the statement was judged to be equivalent, the number of the model heuristic was entered on a tally sheet. Each person also underlined the phrases of each statement on which they based their judgments. If a statement was judged

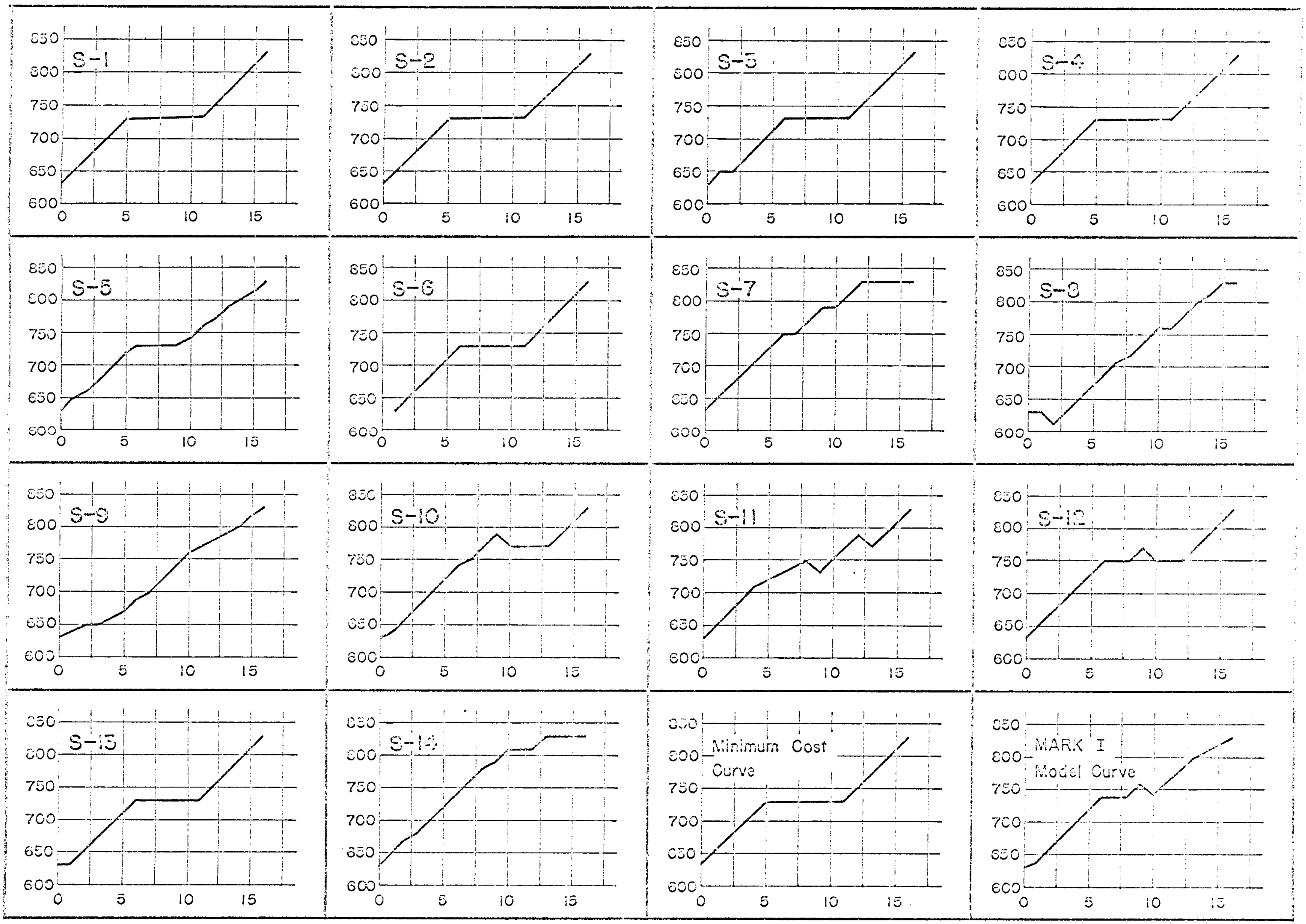


FIGURE 8. SUBJECT TRAJECTORIES OBTAINED FOR PROBLEM 10 IN MARK I TRIALS

TABLE 7. VERBAL STATEMENTS FOR SUB-PROBLEM 10 IN MARK I

Subject	Statement
1	No statement (Subject continued to use the strategy he stated in Sub-problem 2 -- "I have found that there is one speed at which the fuel consumption is zero and the speed is characterized by the fact that on both sides the fuel consumption steadily increases. It appears to be wise to reach this speed at which the consumption is zero as rapidly as possible to maintain the speed of it until the end then dropping or increasing until the final required speed.")
2	It would appear that it would be to your advantage to find a minimum point in fuel consumption and then stay there as long as you can. Be careful to note how many steps you have until you reach your final velocity, how many steps it will take you to reach your final velocity. Then maintain yourself at the low fuel consumption as long as possible and then make the jump to the final velocity. This method seems to be different with different jumps. In other words, you'll get a minimum point at different points, but there always seems to be a minimum point that you can reach. Stay there as long as possible and then make the jump up to the final velocity.
3	Be careful punching the buttons. My punching the buttons wrong costed me an increase of about 15 percent fuel, I think. I'm 100 percent confident that you should be careful about punching the buttons.
4	I'd offer the same advice. I'm 96 percent confident--from Sub-problem 6. I'd advise trying to find a rest stop that will consume little fuel when there is no change in the velocity, holding there, and then approaching the final velocity in the last few tries.
5	No statement. Statement from 9--In this flight you reach a velocity at which the fuel consumption is a minimum. In this case it was less than the final desired velocity and you decreased by one. I'm 75 percent confident that increasing your fuel to the final desired velocity you will conserve the fuel.

TABLE 7. (Continued)

Subject	Statement
6	I found in calculating the distance unless you look at previous results and see how many you go over, the distance does remain the same. Statement from 9--It is almost 100 percent true that to keep your costs at the lowest minimum, find your place and stay there. Calculate how far you are from the zero spot and come back over at the end calculating correctly. Also at the beginning start out at zero and then go to one side or the other side to find out which way the scale will go.
7	No statement
8	No statement
9	I'm confident, 100 percent, I think I'm still following the same pattern. Make the slope very slight.
10	No statement. Statement from 9--After you determine your zero and are using the + zero to keep your costs at a minimum. Plan to use the minimum number of decisions to retain your final velocity like using the majority of +2 or -2 to get to that velocity once you determine the zero points. I'm 100 percent confident.
11	Proceeding on same theory, confidence is up to 95 to 100 percent. Statement from 6--It's only a theory at the present time but seems as you either increase or decrease velocity (and this time decrease) where you reach a point where your fuel consumption is at a minimum and going to one side or the other of that point will cause you to consume more fuel. Therefore by maintaining that speed until the last how many steps it takes to reach the velocity desired, you can conserve fuel and therefore, when you come up to your desired velocity and maintain it at zero. Thereafter it still costs you the same number of units of fuel. The example in this case was 2,300 units and I feel if I come up to that point, say on the fourth step, I could maintain zero velocity constantly for the rest of this problem and still use 23,00 units of fuel.

TABLE 7. (Continued)

Subject	Statement
12	No statement. Statement from 7--Concerning general strategy try to find the point where the fuel consumption is lowest and then keep going until you have the minimum number of possible left that you can build up to your final velocity without having a penalty.
13	No statement. Statement from 9--I'm convinced that it pays to go past your final velocity, not too far, but to bring your fuel consumption down as long as you keep it reasonable so you can bring it back up in the amount of time provided.
14	I would approach the final velocity with control value of 2 until getting within 40 miles of the final velocity and then staying there once to see what the cost is. If the cost is less than 3,200, I would recommend staying there until your final decisions. If it isn't, I would recommend going to a velocity of 810 and staying there because it is cheaper to run there than at the final velocity. And then on your second (next to the last decision) you would use the control value of 2 to drop into the 830.

to be a heuristic, but not one which appeared on the list, it was copied onto the tally sheet. Also, any statements which could not be interpreted were entered on the sheet.

HUMAN CONTROLLER AND THE MARK I MODEL

Comparisons of results obtained from the Mark I model and the experimental studies with human controllers are presented in the following sections. An analysis of the performance as measured by subject median fuel cost is made. This analysis is supplementary to that of the verbal statements, and provides a more detailed context for the verbal results presented later. An analysis of the verbal statements made by a panel of three members is summarized and discussed. The use of the median as a measure of location is a convenience in the development of this context. It is not asserted that the median is an appropriate measure of the performance of the group of subjects. A more detailed analysis of the fuel costs for each individual subject is presented in Appendix B.

Measure of Subject Performance

Before an analysis of performance is presented, we will discuss the sequential structure of Mark I control problems and a relative measure of subject performance. Table 8 shows that the number of decisions available gradually decreases through the sequence of 23 Mark I problems. This was done in order to increase the difficulty of the problems. The table shows that 12 of the problems require an increase from the initial to the final velocity; ten problems require a decrease in this velocity, and one problem requires no change in velocity.

TABLE 8. CLASSIFICATION OF MARK I SUBTRAJECTORIES

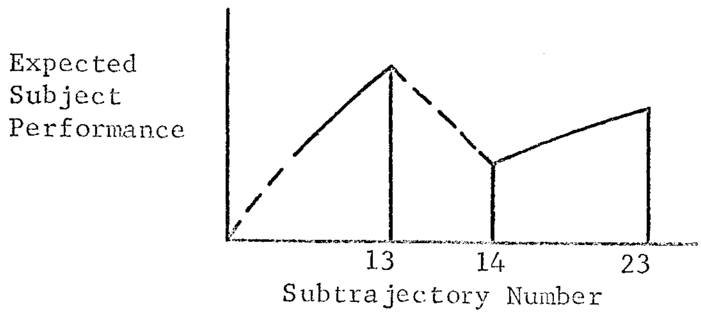
Subtrajectory Number	Number of Decisions	Required Velocity Change	Form of Optimal Trajectory
1	38	+	(1)
2	34	-	(2)
3	30	+	(1)
4	26	-	(2)
5	22	+	(1)
6	18	-	(2)
7	14	+	(1)
8	18	+	(3)
9	17	-	(4)
10	16	+	(1)
11	16	-	(2)
12	16	+	(3)
13	15	-	(4)
14	14	+	(5)
15	13	-	(6)
16	12	+	(3)
17	11	0	(4)
18	10	-	(6)
19	9	+	(5)
20	9	+	(5)
21	9	-	(6)
22	8	+	(5)
23	8	-	(6)

The last column of the table shows six forms of the underlying minimum cost subtrajectory. The form for subtrajectory number 1 shows, for example, that the minimum cost trajectory consists of an initial increase in velocity, followed by a horizontal (constant) velocity, and finally a further increase to the required final velocity. The horizontal portion corresponds to the reference level at which the incremental fuel cost is zero. Examination of this column shows that the first seven problems have the reference level between the initial and final velocity. These forms are the simplest types. If the final velocity is achieved, then at some previous time the reference level must be crossed.

A convenient measure of subject performance for a particular problem is given by the ratio R of the fuel cost obtained with the Mark I model to the subject median fuel cost. Because a control objective consists of minimizing the toal fuel consumed for each problem, small values of both the numerator and denominator of R are desirable. Consequently, the ratio yields a relative measure between the performance of the subjects as a group and the performance of the model. If the subject median cost is must smaller than that obtained with the Mark I model, then R is large and the group performance is good relative to the model performance. Conversely, if the subject median cost is much larger than that obtained with the model, then the group performance is poor relative to the model performance. The behavior of the R-ratio over the set of control problems thus serves as a relative measure of the quality of the group performance. In addition, as discussed later, this same ratio may be associated with group learning.

It was expected that with only moderate attention to the meter readings the subjects would detect the existence of the reference level and would make use of it in the choice of their controls. Moreover, it was expected that the subjects would verbally announce these detected structures in the form of heuristics. The first seven problems were designed to be easy, but were also intended to mislead the subjects into expecting the reference level to be betweem the initial and final velocities. A heuristic based on this expectation would be found to be incorrect in problems 8 and 9 where the reference level lies in the same direction, but beyond the final velocity. A corrected verbal heuristic to meet this situation would need further revision when problem 14 is encountered. In this problem the subjects encounter, for the first time, a problem in which the reference level lies in a direction opposite to that of the final velocity. It was expected that subject performance would drop appreciably at problem 14. Moreover, from problem 14 through problem 23, the subjects would no longer be able to predict the location of the reference level. This would require some trial-and-error behavior at the beginning of each problem in order to determine the direction associated with minimum incremental fuel costs.

The following sketch shows that subject performance was expected to be erratic for the first few problems until the reference level was detected and used. This would result in improved performance through problem 13. A marked drop in performance was expected at problem 14. However, it was expected that performance would improve through the remaining problems, but would not reach the level of the earlier problems because of difficulty in locating the reference level.



<u>Correlation Between Mark I and Subject Median Fuel Costs</u>

To make comparisons of results obtained from the Mark I model and the human controllers, the correlation between Mark I and subject median fuel costs is calculated and plotted. Table 9 shows the median of the cumulative fuel costs for the 14 subjects and the cumulative fuel cost obtained for the Mark I model. These fuel costs include the miss-distance penalties for the subjects. For the simulation model these penalties were zero because the model always obtained the desired final velocity for the Mark I control problems.

Figure 9 shows the cumulative fuel cost as a function of subtrajectory number for the median of the 14 subjects and for the Mark I model. This plot is based on the numbers shown in Table 9. It is apparent that a high correlation exists between these plots. A rather large difference in fuel consumption occurs at problem 14, as expected.

Figure 10 shows the same results as the preceding figure. With the cumulative fuel costs plotted on a logarithmic scale, the high correlation is more clearly seen. Although the correlation is high the percentage deviation is quite large for some problems. This is shown by the next plot.

TABLE 9. SUBJECT MEDIAN COST AND MARK I SIMULATION FOR EACH SUBTRAJECTORY

Subtrajectory Number	Kilo Units of Fuel	
	Median Subject Cost*	Mark I Cost
1	35.150	83.900
2	30.400	39.900
3	1.635	1.830
4	76.000	84.500
5	23.850	11.400
6	9.101	7.106
7	8.046	6.166
8	95.250	124.400
9	5.180	10.650
10	21.575	25.300
11	128.950	153.800
12	27.225	32.900
13	65.200	94.500
14	80.500	37.000
15	2.175	1.020
16	17.600	14.000
17	11.700	10.080
18	22.600	21.800
19	1.330	1.180
20	44.400	33.600
21	20.650	23.250
22	4.024	4.579
23	41.700	47.700

*Median of 14 Subject Costs

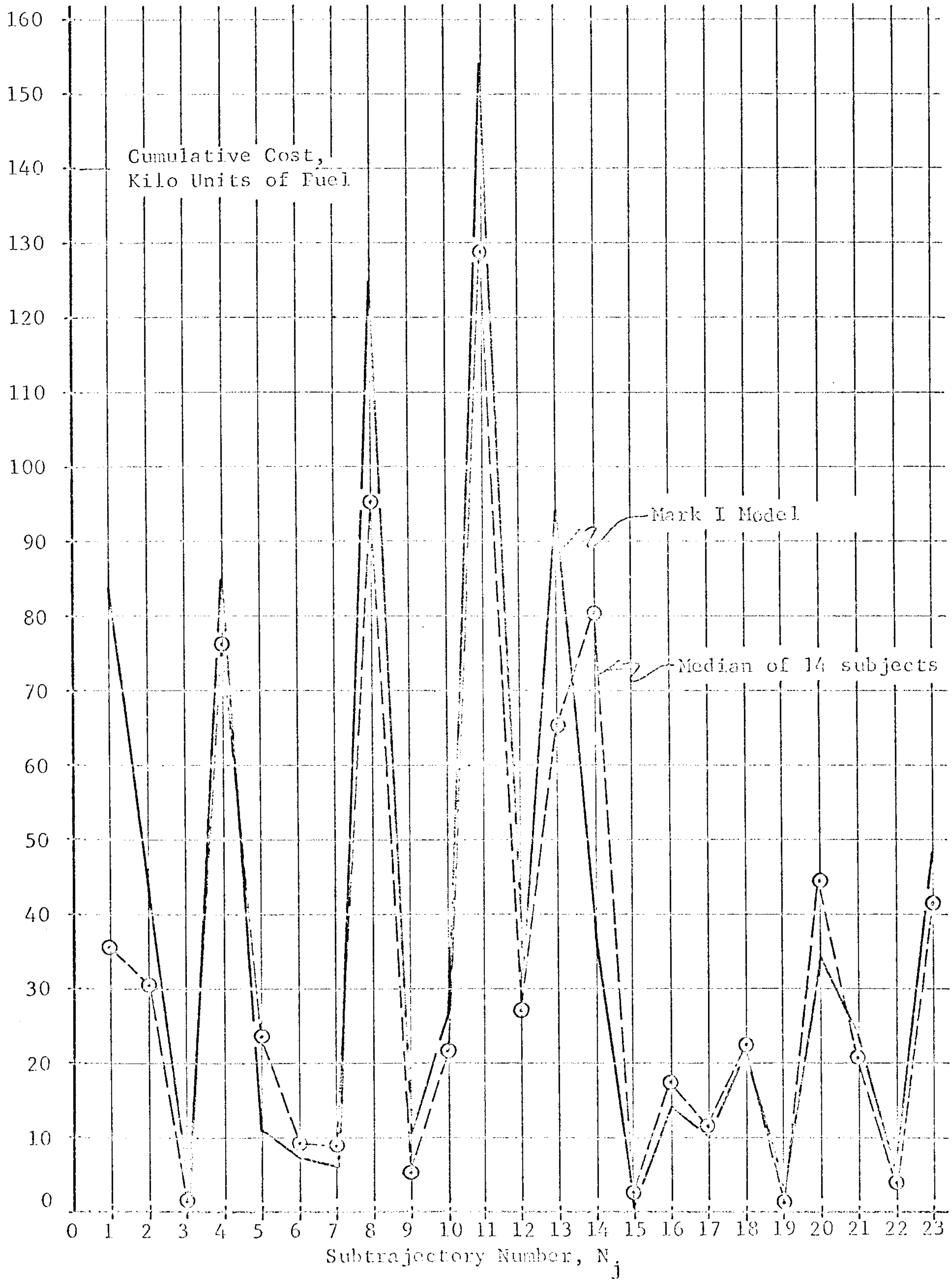


FIGURE 9. SUBJECT MEDIAN COST AND MARK I MODEL COST

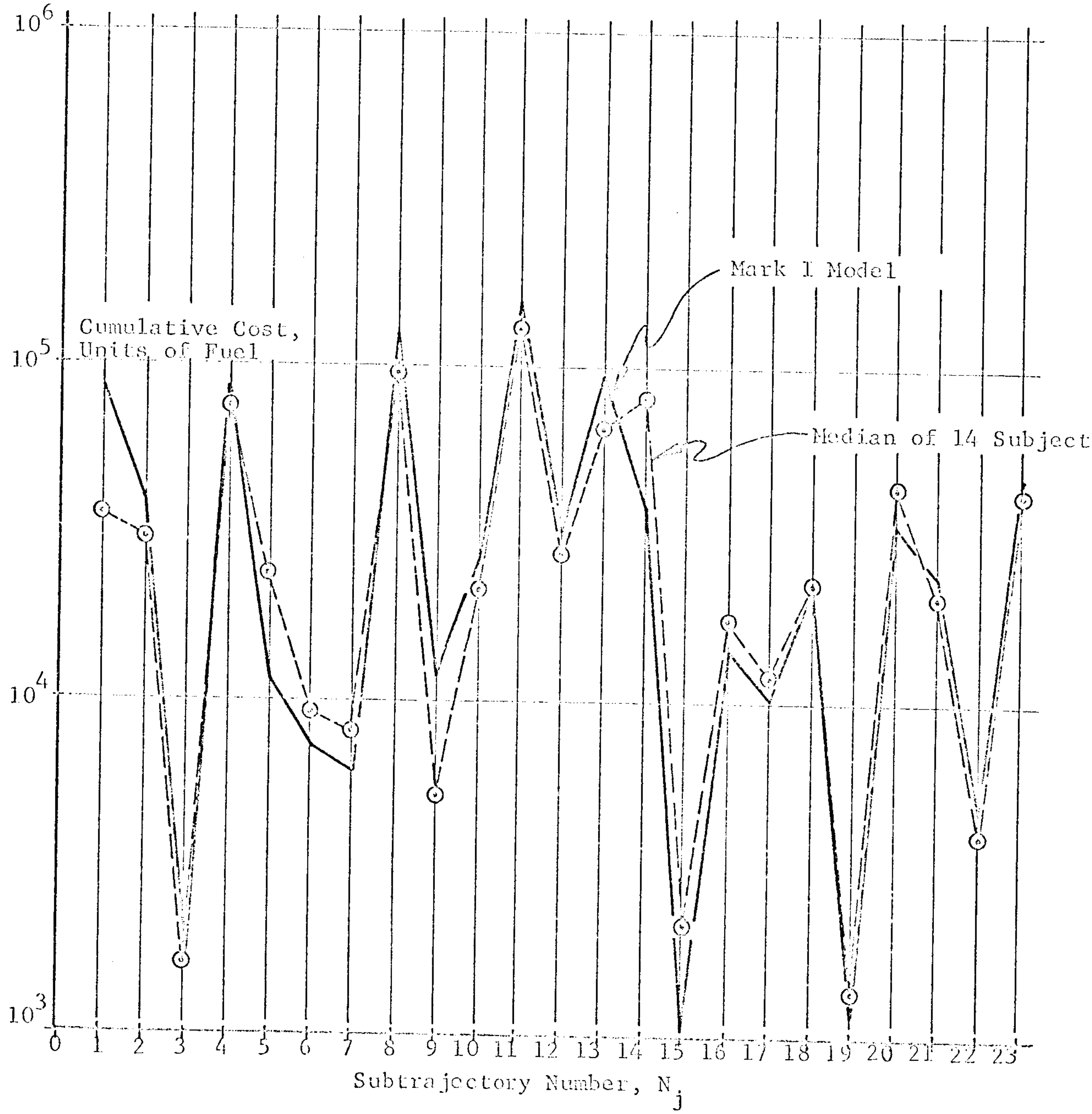


FIGURE 10. SUBJECT MEDIAN COST AND MARK I MODEL COST

Figure 11 shows a plot of the percentage deviation given by

$$D = \frac{\text{Subject Median Cost - Mark I Cost}}{\text{Mark I Cost}} \times 100$$

as a function of subtrajectory. The highest percentage deviation, 117.6 percent occurs at problem 14; the minimum deviation, 3.7 percent, occurs at problem 18. The average of the absolute value of the percentage deviation is found to be approximately 34 percent. Since this includes problems 14 and 15, it gives a conservative measure as the upper bound to the percentage deviation. A lower bound is obtained by taking those problems where the learning is expected to be nearly complete: Problems 10, 11, 12, 13, and problems 20, 21, 22, and 23. The average absolute deviation for these two sets of problems is less than 20 percent.

Figure 12 shows a scatter diagram of subject median cost versus the Mark I model cost on a log-log scale. The plot suggests a linear correlation exists between these measures.

Figure 13 shows a regression line fitted to the scatter diagram. The equation of the regression line is give by

$$\log_{10}(\text{Subject Median Cost}) = 0.525 + (0.877)\ \log_{10}(\text{Mark I Cost})$$

The figure also shows 95 percent confidence limits for the regression line. It is seen that these limits easily contain the ideal regression line (dashed line) corresponding to a perfect correlation. Thus, the observed data do not reject the hypothesis of a perfect correlation between the model fuel costs and the subject median fuel costs.

$$D = \frac{\text{Subject Median Cost -- Mark I Cost}}{\text{Mark I Cost}} \times 100$$

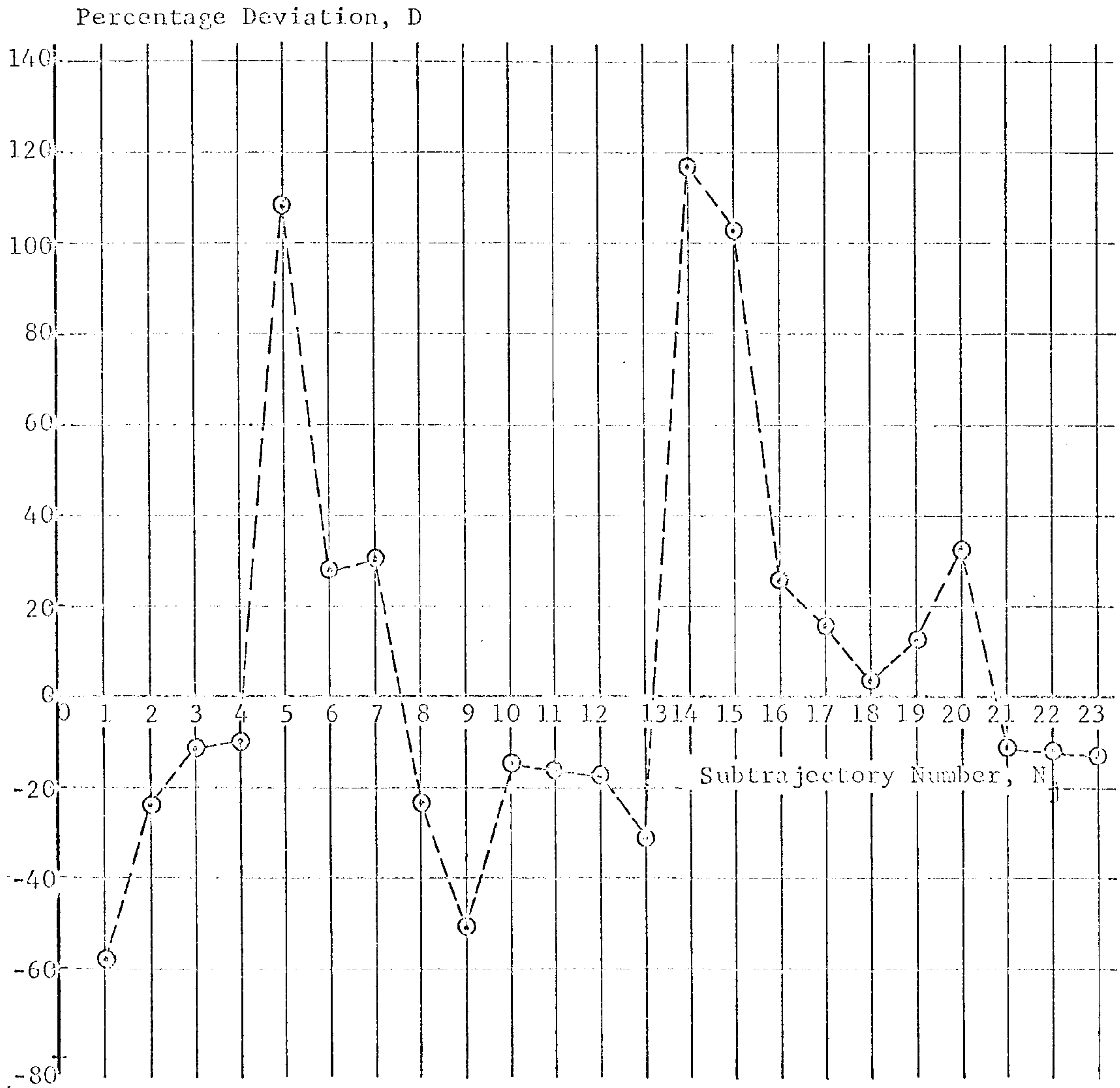


FIGURE 11. PERCENTAGE DEVIATION BETWEEN SUBJECT MEDIAN COST AND MARK I MODEL COST

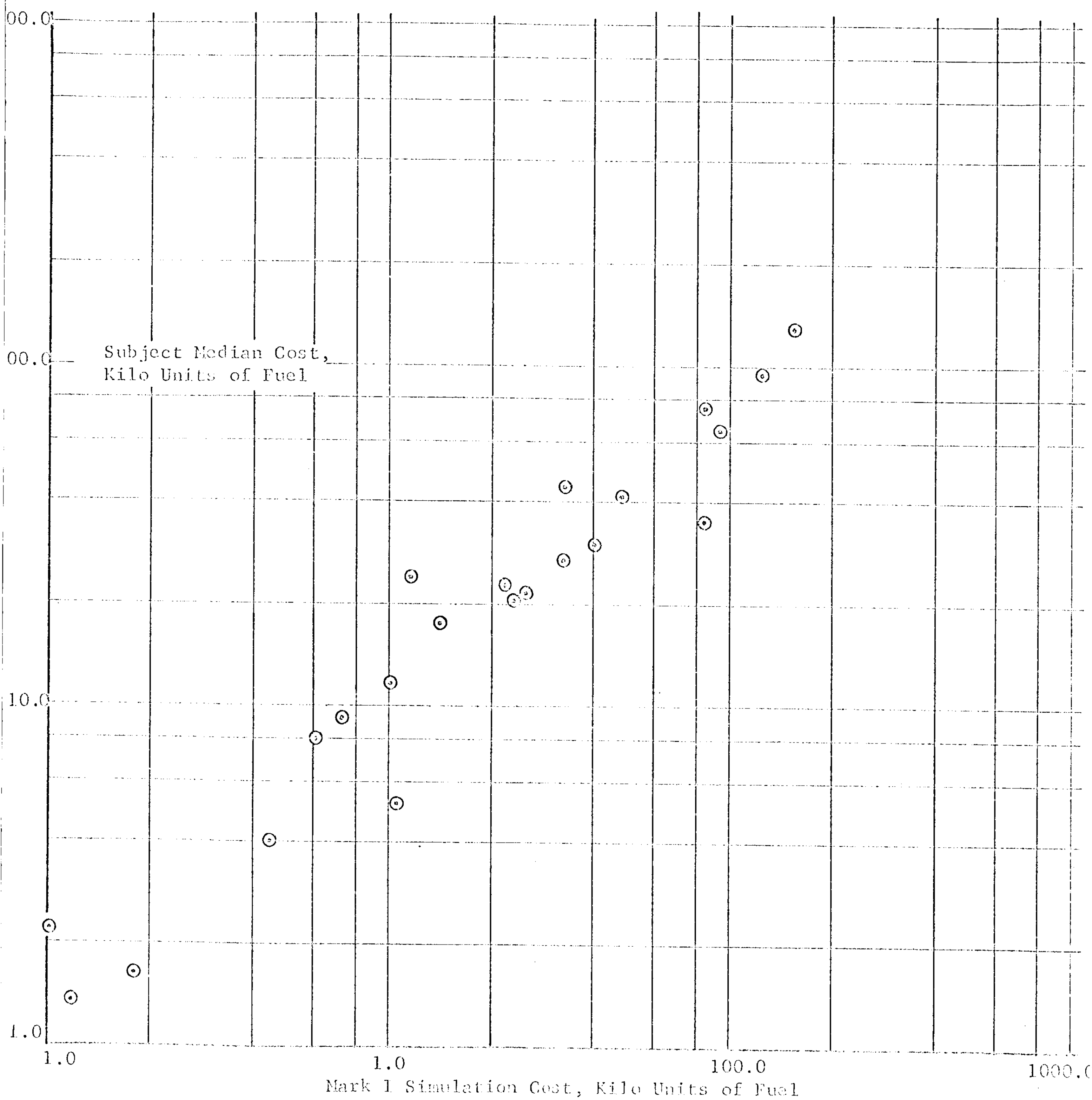


FIGURE 12. SCATTER DIAGRAM OF SUBJECT MEDIAN COST VERSUS MARK I SIMULATION COST

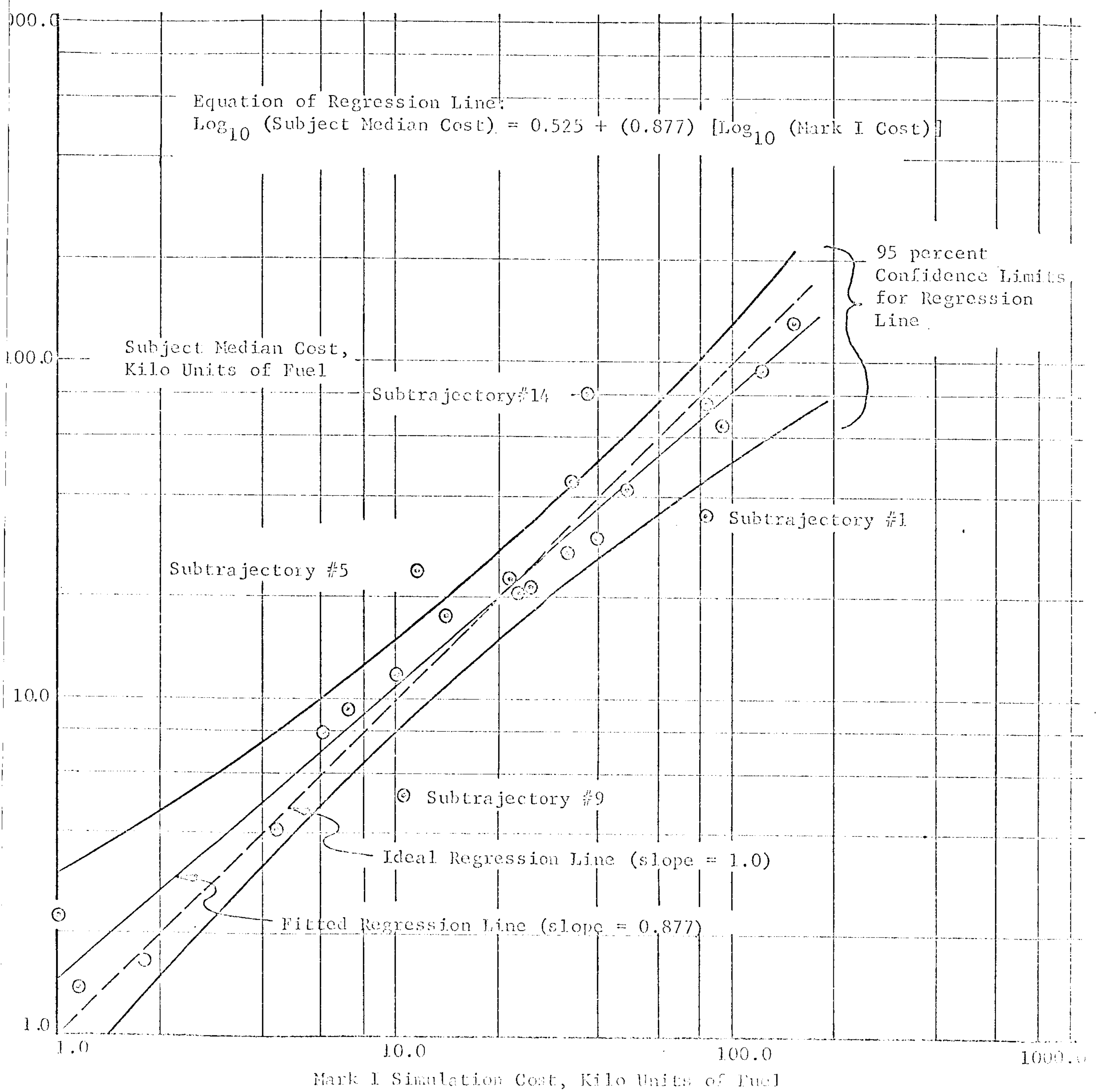


FIGURE 13. REGRESSION LINE FITTED TO SCATTER DIAGRAM FOR MARK I EXPERIMENT

The plot shows that the subject median cost was high relative to the simulation cost for subtrajectories 5 and 14. As noted earlier, the high subject cost for subtrajectory number 14 was expected. In the case of subtrajectory number 5,the reference level was close to the initial velocity and relatively far from the final velocity. Consequently, the problem of when to leave the reference level was somewhat difficult. Most subjects had not as yet learned to deal effectively with this situation. The model costs for subtrajectories 1 and 9 are seen to be high relative to the subject median costs. This result was expected for subtrajectory number 1 because the model begins the first problem by using the probing mode of control and applying, in succession and regardless of cost, every alternative control available to the system. This is done to get some information on the effect of each control. This procedure was not used by the subjects. The relatively poor model performance on subtrajectory number 9 resulted from the use of y = -1 under gradient control for most of the problem. Although the repeated use of this control was warrented by the decreasing incremental fuel cost, the rate of decrease was too slow to reach the reference level before the initiation of terminal control. Some subjects, in contrast used y = -2 and attained the reference level where the incremental fuel cost was zero.

An examination of the deviations of the observed points from the fitted regression line was made to determine whether these deviations could be regarded as normally distributed. The results showed a distribution which is symmetric but more concentrated about the mean than the normal distribution. For this reason the following significance tests must be regarded as approximate tests.

The slope of the fitted regression line is 0.877. An approximate t-test of the significance of the difference between this observed slope and an ideal slope of 1.00 gives a computed value of t equal to 1.18 with 21 degreees of freedom. The corresponding tabulated value of t at the 0.95 fractile is equal to 2.08. Thus, no statistical significance is shown at the 5 percent significance level between the observed slope of 0.877 and the hypothetical slope of 1.00.

The correlation coefficient between the logarithm of the subject median cost and the logarithm of the Mark I model cost is found to be equal to 0.96. The correlation coefficient of the untransformed costs is found to be 0.92.

The above analysis suggests a measure of subject learning. Figure 14 shows a plot of the ratio, R, of the fuel cost using the Mark I model to the median fuel costs obtained for the 14 subjects:

$$R = \frac{\text{Fuel Cost for Mark I Model}}{\text{Median Fuel Cost for 14 Subjects}} .$$

Because the objective in the problems is to minimize fuel costs, it is seen that when $R < 1$ the Mark I model performance is better then the subject median performance. Conversely, when $R > 1$ the subject median performance is better than the Mark I model performance.

The plot suggests that subject median performance improves for problems 5 through 13. A marked drop in performance occurs at problem 14 but further improvement occurs until the end of the set. In general, the Mark I model does not learn. Only the memory limit changes as a function of experience. This value became equal to 1 at subtrajectory 8 and remained at this value for the rest of the sequence. Thus, the improved performance shown by the subject median cost can be attributed to human learning. As noted earlier, human learning was expected to occur in these two segments.

R < 1: Mark I Cost Lower

R > 1: Subject Median Cost Lower

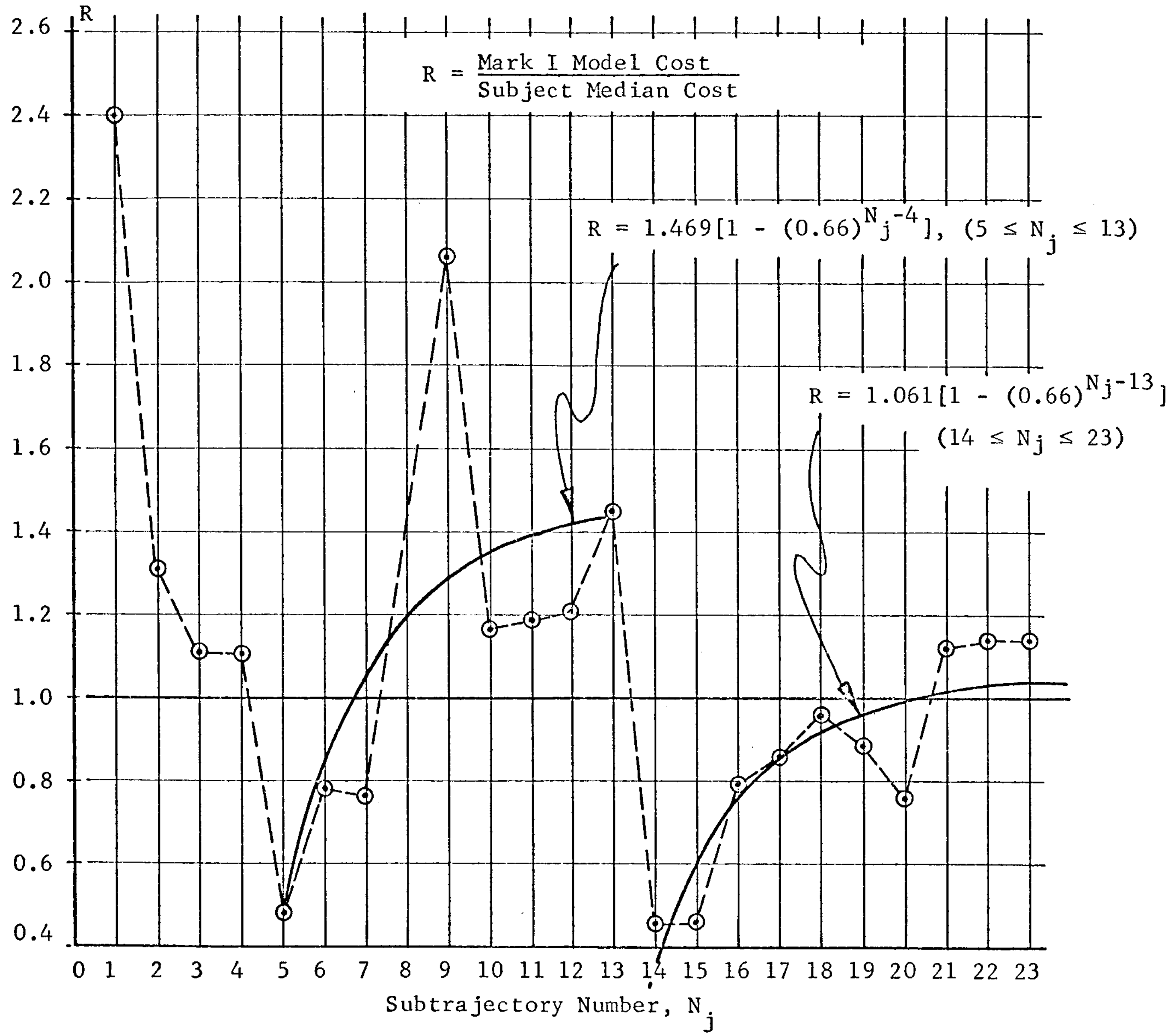


FIGURE 14. LEARNING CURVES FOR SUBJECTS IN MARK I TRIALS

Learning curves of the following form were fitted to the data:

$$R_k = R^* (1 - r^k) \quad ,$$

where R_k represents the value of R at the k^{th} subtrajectory. R* and r are fitted constants. This expression is a discrete form of Hull's empirical learning curve.(15) The constants were fitted by minimizing a quadratic performance index expressed as the sum of the squared deviations:

$$Q = \sum_{k=1}^{n} [R_k - R^*(1 - r^k)]^2 \quad .$$

It is easy to show that for a fixed value of r, Q is minimized by the following value of R*:

$$R = \frac{\sum_{k=1}^{n} R_k(1 - r^k)}{\sum_{k=1}^{n} (1 - r^k)^2} \quad .$$

With this value of R* as fixed, the value of r was varied by trial to improve the minimum. By such iterations the following equations were obtained:

$$R = 1.469[1 - (0.66)^{N_j - 4}] \quad , \quad 5 \le N_j \le 13$$

$$R = 1.061[1 - (0.66)^{N_j - 13}] \quad , \quad 14 \le N_j \le 23 \quad .$$

The asymptotic values of R are seen to be 1.469 and 1.061 for the early and late learning curves. As expected, the learning curve for the late segment of problems asymptotes to a lower level than that for the early learning curve. The value of 0.66 for both curves is not the optimum value, but differs from the optimum by a small percentage. It was found that the same value of r could be used for the Mark II analysis so this compromise seemed desirable.

Some Comparisons Based on Chi-Square Tests

To make further comparisons, chi-square tests were performed. Table 10 shows the computations for a χ^2 - test. As a test hypothesis, it is supposed that, based on total fuel consumption, the Mark I model may be regarded as a "typical" subject. Such a typical subject would have the property that on any given subtrajectory the typical subject would have a cost better than half of the real subjects and worse than the other half of the real subjects. With 14 real subjects, the expected number of subjects that would be expected to have costs greater than the Mark I costs would be 7. Thus, we may compare the observed and expected number of subjects having costs greater than the Mark I costs. The third column of the table shows the difference, Δ, between the observed and expected numbers. Subtrajectory 14 is a special case. Because of the expected degradation of the performance of the subject in this subtrajectory, it would appear desirable to use 14 as the expected number of subjects having costs greater than the Mark I costs. If this is done, then the difference between the observed and expected number is zero, as shown in parentheses.

The square of the difference is shown in the last column of the table, and the totals are found to be 254 or 205, depending on whether the expected number for subtrajectory 14 is taken to be 7 or 14. In the first case, the computed value of χ^2 is given by $\chi^2 = 254/7 = 36.3$. The tabulated 95 percent fractile of the χ^2-distribution with 23 degrees of freedom is found to be 35.2.

TABLE 10. COMPUTATIONS FOR A CHI-SQUARE TEST

Subtrajectory Number	Observed Number of Subjects With Costs Greater Than Mark I Cost	Observed Number Minus Expected Number, Δ	Δ^2
1	3	-4	16
2	4	-3	9
3	6	-1	1
4	6	-1	1
5	12	5	25
6	8	1	1
7	10	3	9
8	3	-4	16
9	2	-5	25
10	4	-3	9
11	3	-4	16
12	5	-2	4
13	3	-4	16
14	14	7(0)	49(0)
15	11	4	16
16	8	1	1
17	8	1	1
18	9	2	4
19	9	2	4
20	10	3	9
21	4	-3	9
22	5	-2	4
23	4	-3	9
	151		254(205)

Thus, in the first case it is suggested that the differences between the Mark I model and the subjects are statistically significant at the 5 percent level of significance. However, in the second case, with an expected number of 14, the computed value of χ^2 is given by $\chi^2 = 205/7 = 29.3$.

Since 29.3 is less than 35.2, in this case it is found that the observed and expected results are within statistical agreement.

A similar χ^2-test can be made as follows. Under the test hypothesis that the Mark I model is a "typical" subject, a given subject should perform better than the model on half of the subtrajectories.

Table 11 shows the computations for such a χ^2-test. The expected number was taken equal to $23/2 = 11.5$. The computed value of χ^2 is then found to be given by $\chi^2 = 327.50/11.5 = 28.5$. The tabulated value of the 95 percent fractile of the χ^2-distribution with 14 degrees of freedom is equal to 23.7. Thus, the test shows that a given subject does not perform better than the model on half of the subtrajectories.

The table shows that Subject 9 deviated widely from the expected value. The experimenter recorded the following remark immediately after the trials with this subject:

> "Subject 9 demonstrated extremely rigid behavior. He had good involvement but completely ignored the cost function even though the subject was told to minimize fuel consumption several times."

With this justification the same χ^2-test may be applied to the remaining 13 subjects. The sum of the required deviations is found to be 217.25 and the computed value of χ^2 is given by $\chi^2 = 217.25/11.5 = 18.9$. The 95 percent fractile of the χ^2-distribution with 13 degrees of freedom is found to be 22.4. Thus, with

TABLE 11. COMPUTATIONS FOR A CHI-SQUARE TEST

Subject Number	Observed Number of Subproblems for Which Subject Costs Exceeded Mark I Costs	Observed Minus Expected, Δ	Δ^2
1	7	-4.5	20.25
2	12	0.5	0.25
3	4	-7.5	56.25
4	10	-1.5	2.25
5	8	-3.5	12.25
6	10	-1.5	2.25
7	16	4.5	20.25
8	8	-3.5	12.25
9	22	10.5	110.25
10	9	-2.5	6.25
11	10	-1.5	2.25
12	7	-4.5	20.25
13	9	-2.5	6.25
14	19	7.5	56.25
	151		327.50

Subject 9 excluded, the hypothesis that the model performs better on half of the subtrajectories for a given subject is consistent with the data at the 5 percent level.

As another test of how well the model typified the humans, the following test was conducted. Each Mark I subtrajectory for each subject and the model was individually plotted, thus producing 15 plots for each of the 23 subtrajectories. The 15 plots for each subtrajectory then were arranged in a random sequence and given to three persons not previously associated with the project. These persons were told that the trajectories had been generated by 14 humans and one machine and that their task was to select which trajectory had come from the machine. Out of the 69 selections (23 subtrajectories for three persons) there occurred only one correct match. There were, however, several matching selections among the persons. On subtrajectory 1, two persons selected the plot from Subject 5. On subtrajectory 8, two of the persons selected Subject 13's plot. On subtrajectories 9, 17, 19, 20, and 21, all three persons chose the trajectories from Subject 9. On problem 13 and 14, all persons chose Subject 7's plots. By chance, there should have occurred four or five correct selections out of the 69 possibilities. Since only one correct selection was obtained, it appears that it was not possible for naive persons to visually select out the model's plots from the humans'. Furthermore, it appears that some subjects, especially 7 and 9, differed more from the other subjects than did the model. Subject 9, it will be recalled was the subject who used a straight line approach for all 23 of the subtrajectories, thus it is not surprising that his plots would be selected.

Analysis and Evaluation of Verbal Statements

The verbal statements made by the subjects for each Mark I subtrajectory were recorded on tape. A typed list of these statements, together with a list of the possible model heuristics, were presented to a panel of three members. Two of these members had no previous connection with the research. The third member had served as the experimenter. Each member independently decided whether a subject's statement was equivalent to any of the possible heuristics. If a match was obtained, the panel member recorded the number of the model heuristic in accord with the numbering given in Table 1a. If no match was obtained, the panel member wrote down the statement or phrases made by the subject. In either case the evidence for the panel member's decision was underlined on the typed copy.

In the analysis of the results, it was further assumed that if a subject made no statement, then his last stated heuristic was still in force. Even with this simplification the analysis was not neat. In many instances the subject would elaborate on previous strategies, or make new observations of fact that were correct but did not appear to change his strategy. Wide discrepancies among the panel members' judgments were then openly discussed and generally resolved. The most forceful criterion in making these resolutions was the following. Unless the statement, or phrase, indicated <u>how</u> a control value <u>should be</u> selected, then it was not a heuristic, and no change in the previous heuristic was indicated.

The results of the analysis are presented as follows. Table 12 summarizes the results and shows the computation of the conditional probability

TABLE 12. COMPUTATION OF THE CONDITIONAL PROBABILITY THAT A SUBJECT'S HEURISTIC WILL MATCH THAT OBTAINED BY MARK I SIMULATION

Subtrajectory Number	Mark I Heuristic No.	Number of Subjects Having Same Heuristic as Mark I (1)	(2)	(3)	Total, T	Conditional Probability, T/(3)(14)
1	3	6	4	3	13	0.31
2	3	6	9	4	19	0.45
3	3	7	10	5	22	0.52
4	3	8	10	6	24	0.57
5	3	11	11	8	30	0.71
6	3	10	11	9	30	0.71
7	3	9	10	9	28	0.67
8	3	11	12	12	35	0.83
9	None	--	--	--	--	--
10	3, 4	12	12	12	36	0.86
11	None	--	--	--	--	--
12	3, 4	12	11	11	34	0.81
13	None	--	--	--	--	--
14	3	11	12	11	34	0.81
15	3	13	12	11	36	0.86
16	3	13	12	11	36	0.86
17	3	13	12	11	36	0.86
18	3, 4	12	11	12	35	0.83
19	3	12	11	11	34	0.81
20	None	--	--	--	--	--
21	None	--	--	--	--	--
22	None	--	--	--	--	--
23	2	12	11	11	34	0.81

that a subject will have the same heuristic as that obtained by the Mark I model. Column 2 lists the heuristics evolved by the model in accord with Table 6. Columns 3, 4, and 5 give the results obtained from the three panel members. The totals in Column 4 are divided by the product of the number of subjects, 14, and the number of panel members, three, to obtain the estimate of the conditional probability given in the last column.

Shown in Figure 15 is a plot of the number of subjects having the same heuristic as the Mark I model. The lower curve is obtained by using the minimum number of heuristic matches given by any one of the three panel members. The upper curve is similarly obtained by using the maximum number of matches given by any one of the three panel members. The intermediate curve is the average conditional probability obtained from the preceding table.

The wide limits for the initial problems can be associated with the generally "fuzzy" statements made by the subjects and with the disagreements of the panel members over the meanings of these statements. As the number of the subtrajectory increases, it is seen that the panel members are more in agreement as shown by the convergence of the upper and lower limits. Figure 15 also points out that the average number of matches increases rapidly over the initial subtrajectories. The average conditional probability over the first seven subtrajectories is equal to 0.56; the average conditional probability taken over the remaining subtrajectories is equal to 0.83. A conditional probability of 0.786 corresponds to a match of heuristics for 11 out of 14 subjects. Ninety-five percent confidence limits for a probability estimated by the fraction 11/14 are given by the interval (0.49, 0.95).

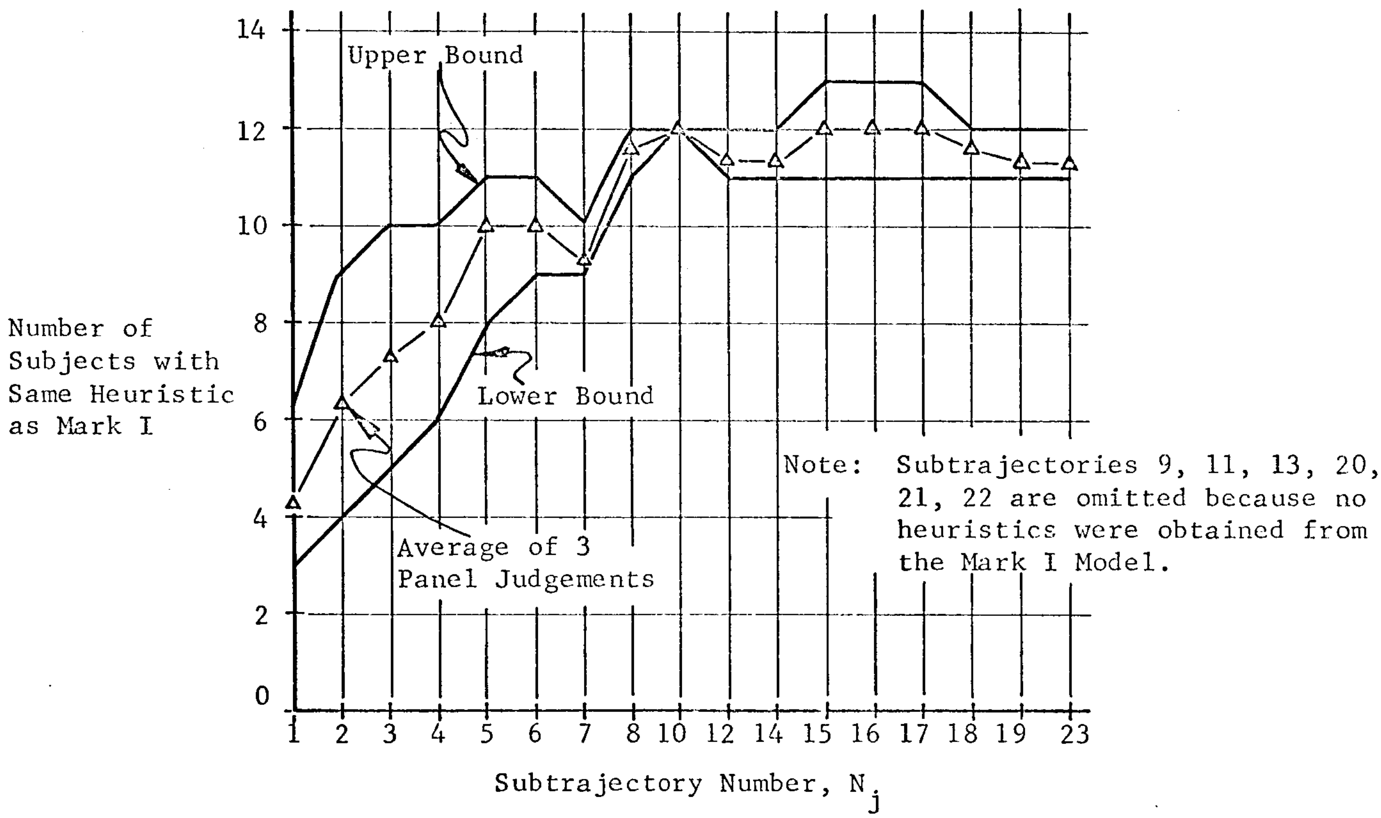


FIGURE 15. NUMBER OF SUBJECTS WITH THE SAME HEURISTIC AS THE MARK I MODEL

HUMAN CONTROLLER AND THE MARK II MODEL

The results obtained from the Mark II model and the experimental studies with human controllers are compared and analyzed in the following sections. The performance as measured by subject median fuel costs is analyzed. Correlation between Mark II model and the subject median fuel costs is determined. The analysis and evaluation of the verbal statements made by three panel members are discussed.

Correlation Between Mark II and Subject Median Fuel Costs

The comparisons of results obtained from the Mark II model and the human controllers are made via correlation analysis. Shown in Figure 16 is the plot of fuel cost as a function of subtrajectory number for the median of the 14 subjects and for the Mark II model. This plot is based on the costs shown in Table 13. Because of the time constants involved in the trajectories of the Mark II model, these results may be re-grouped according to the time constants.

Figure 17 shows the same information as that given in Figure 16 except that the results are grouped according to the three time constants. This figure suggests a high correlation and close agreement for the small time constant equal to 2.4. For the intermediate time constant, a general correlation is preserved, but rather large deviations between the costs are also developed. For the large value of the time constant, the curves show large differences among the costs. In general, these results suggest that the validity of the Mark II model in predicting fuel costs depends on the value of the time constant.

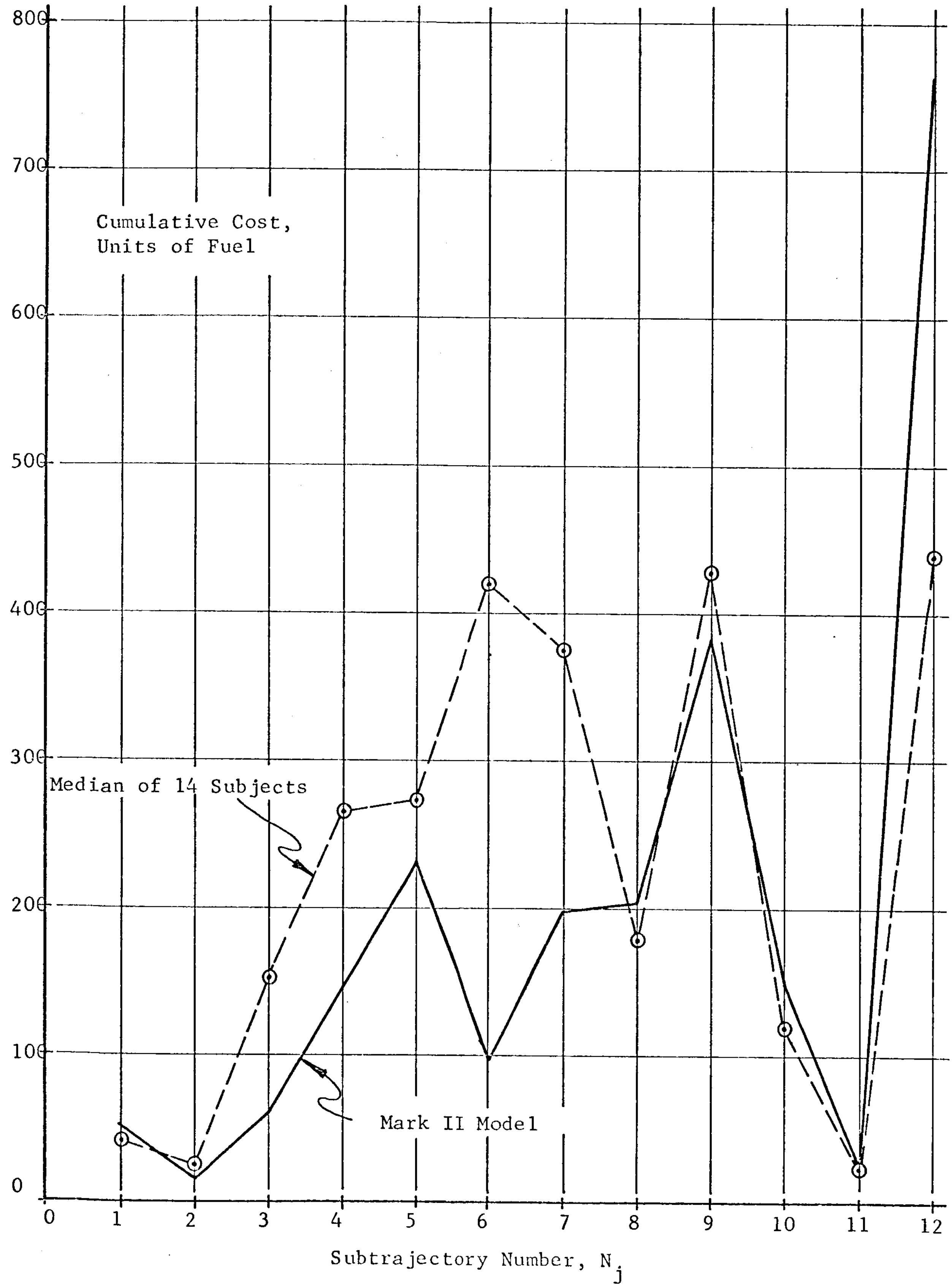


FIGURE 16. SUBJECT MEDIAN COST AND MARK II MODEL COST

TABLE 13. MEDIAN SUBJECT COST AND MARK II SIMULATION COST FOR EACH SUBTRAJECTORY

Subtrajectory Number	Units of Fuel	
	Subject Median Cost*	Mark II Cost
1	42.77	52.40
2	27.12	16.38
3	151.63	59.79
4	265.69	147.53
5	274.00	232.41
6	418.77	97.78
7	373.83	197.43
8	177.91	204.20
9	425.43	382.40
10	119.79	148.66
11	22.80	24.13
12	439.56	761.43

*Median of 14 Subject Costs

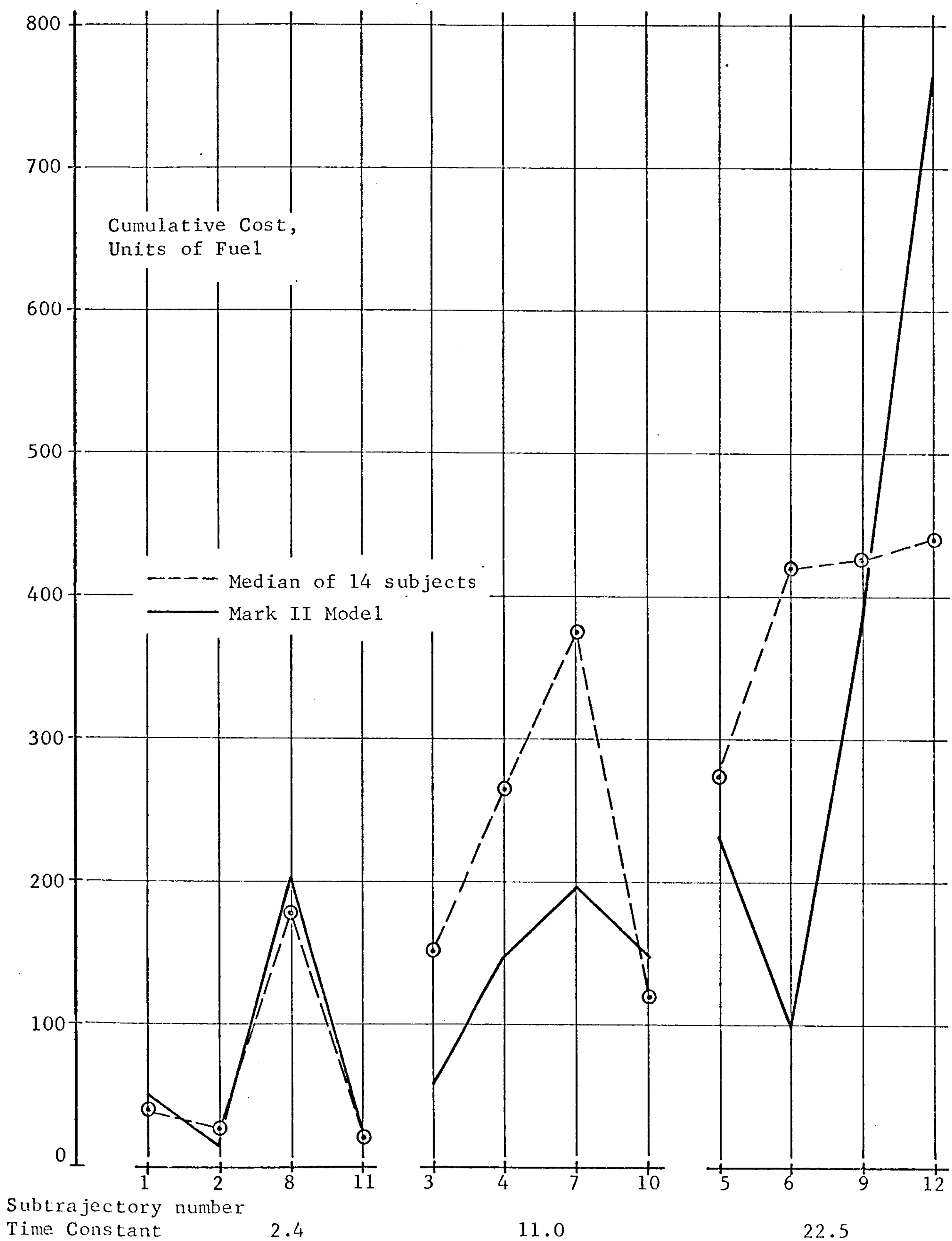


FIGURE 17. SUBJECT MEDIAN COST AND MARK II MODEL COST GROUPED ACCORDING TO TIME CONSTANT

For small values the agreement is good. However, the agreement degrades for increasing values of the time constant.

Shown in Figure 18 is a plot of the percentage deviation between subject median cost and the Mark II cost as a function of subtrajectory and grouped according to the time constant. In general, the results show increasing percentage deviations with increasing time constant with the largest deviation exceeding 300 percent.

A scatter diagram of subject median cost versus the Mark II cost on log-log scales is constructed and plotted in Figure 19. The scatter is seen to be appreciably greater than that shown for the Mark I model in Figure 12.

Plotted in Figure 20 is a regression line fitted to the scatter diagram of Figure 19. The equation of the regression line is given by

$$\log_{10}(\text{Subject Median Cost}) = 2.188 + (0.845)\log_{10}(\text{Mark II Cost})$$

The figure also shows 95 percent confidence limits for the regression line. It is seen that these limits contain the ideal regression line (dashed line) corresponding to a slope of 1.0. As in the case of the Mark I models, the data do not reject the hypothesis of a perfect correlation.

The above analysis leads to a measure of subject learning. Figure 21 illustrates a plot of the ratio,

$$R = \frac{\text{Fuel Cost for Mark II Model}}{\text{Median Fuel Cost for 14 Subjects}}$$

with values of R less than 1 associated with better performance of the Mark II model. In the first six problems the subjects were exposed to two problems of each of the three time constants. After Problem number 6 no new time constants

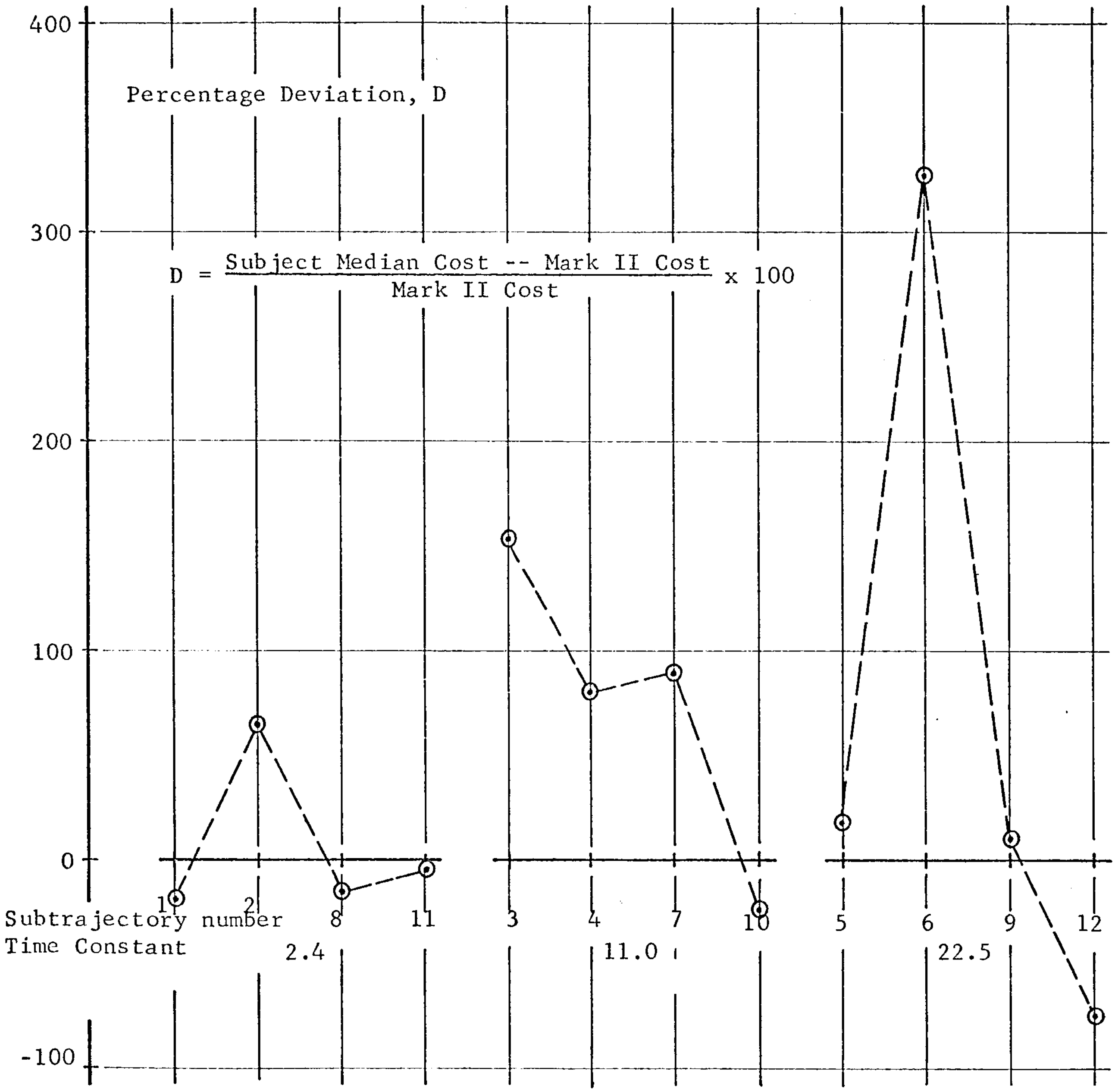


FIGURE 18. PERCENTAGE DEVIATION BETWEEN SUBJECT MEDIAN COST AND MARK II MODEL COST GROUPED ACCORDING TO TIME CONSTANT

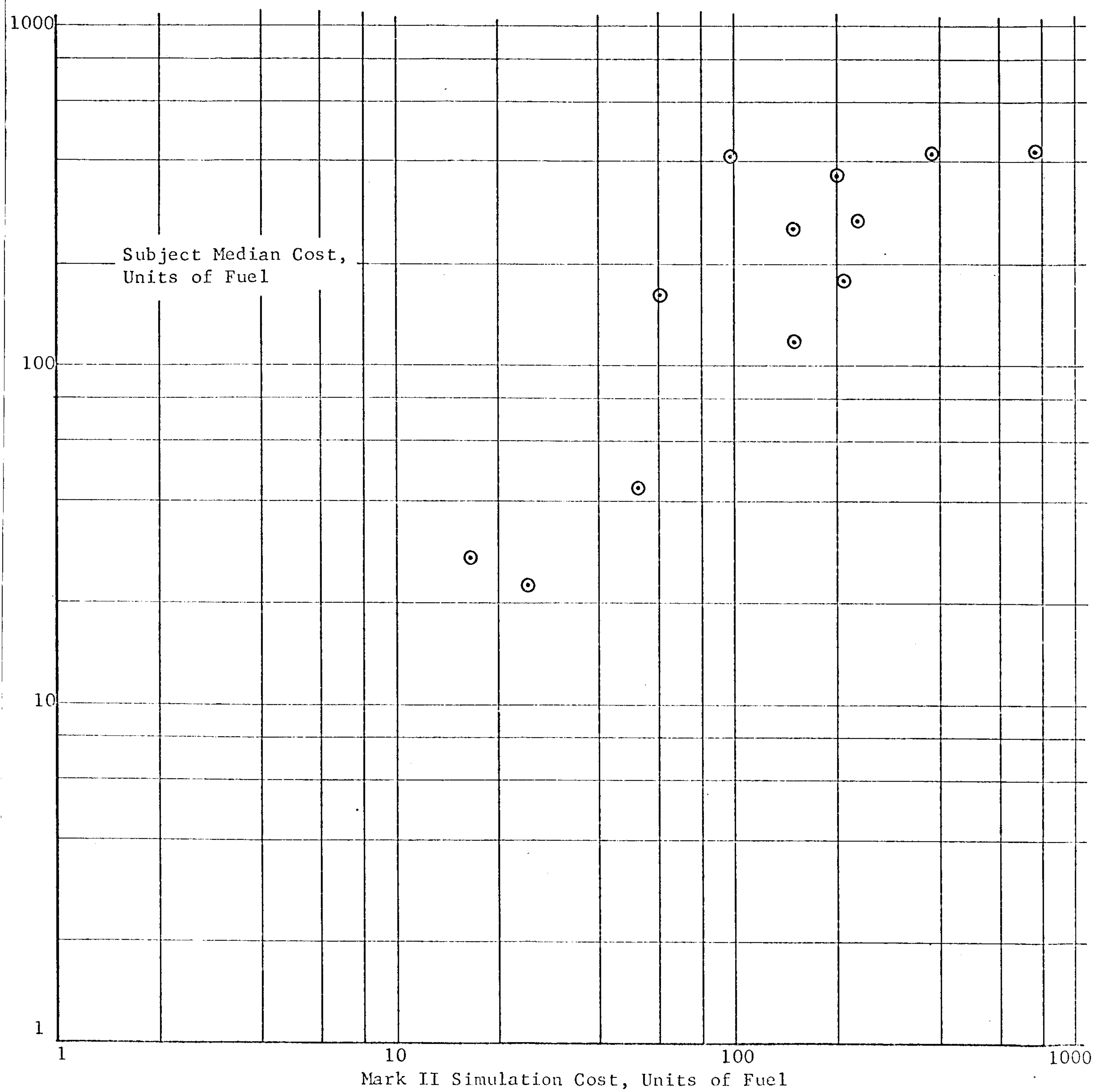


FIGURE 19. SCATTER DIAGRAM OF SUBJECT MEDIAN COST VERSUS MARK II SIMULATION COST

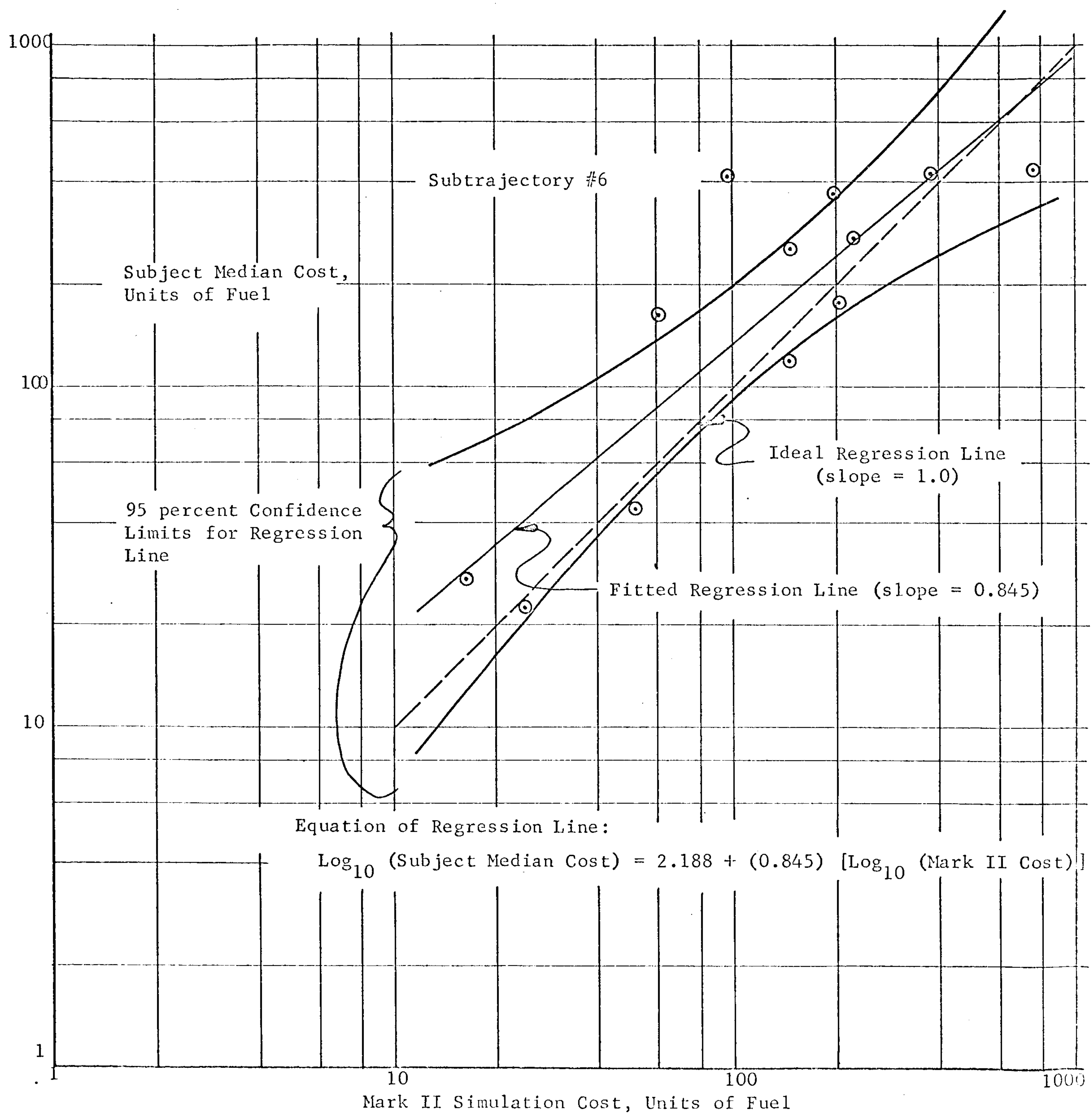


FIGURE 20. REGRESSION LINE FITTED TO SCATTER DIAGRAM FOR MARK II EXPERIMENT

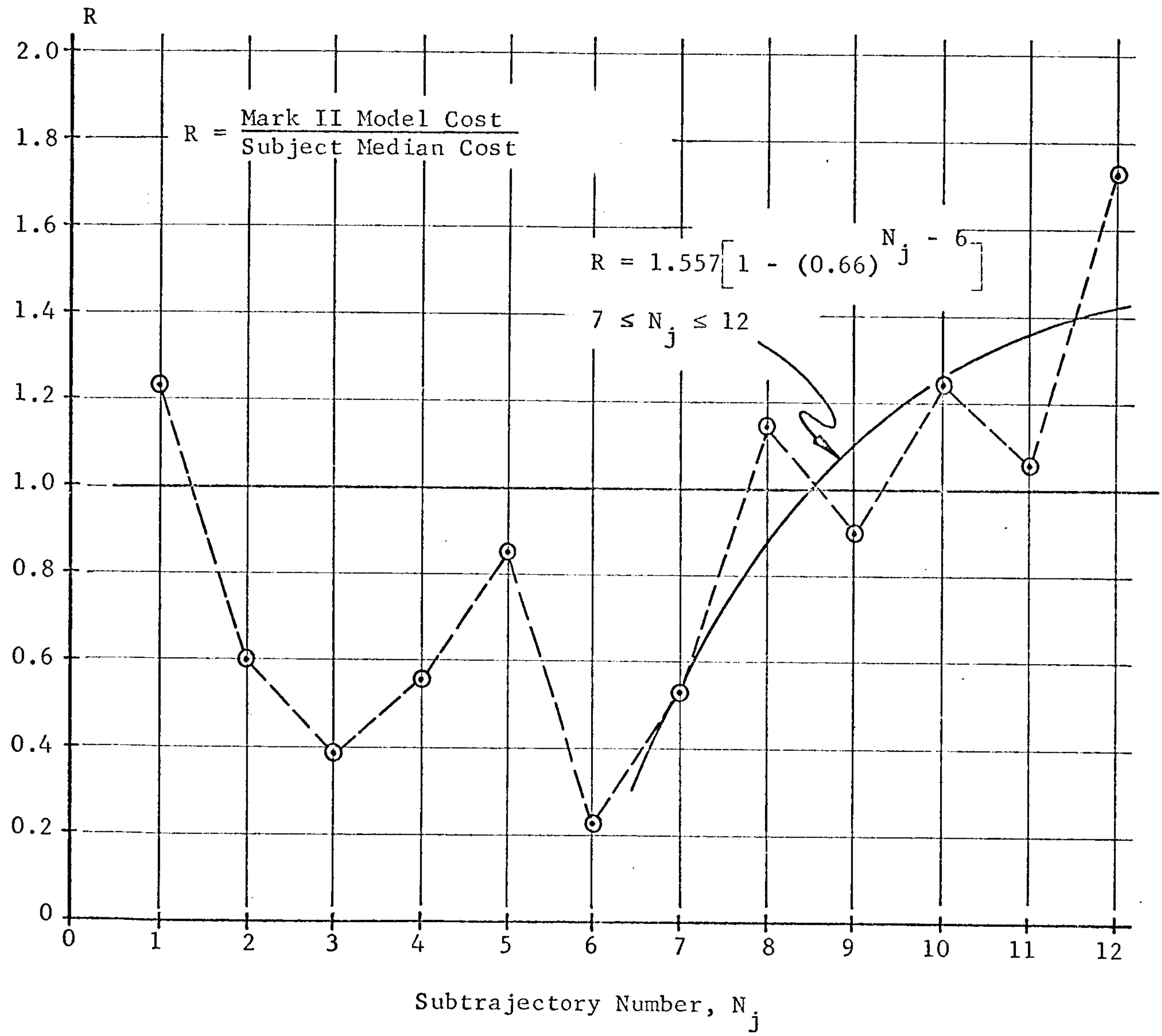


FIGURE 21. LEARNING CURVE FOR SUBJECTS IN MARK II TRIALS

were encountered. Thus, Hull's empirical learning curve was fitted to the last six problems to obtain

$$R = 1.557[1 - (0.66)^{N_j - 6}] \quad 7 \le N_j \le 12$$

The value of $r = 0.66$ was found to be satisfactory for both the Mark I and Mark II learning curves.

Analysis and Evaluation of Verbal Statements

The analysis of verbal statements is studied in this section. Table 14 summarizes the results of analyzing the verbal statements for the Mark II control problems. Column 2 lists the heuristics evolved by the model; columns 3, 4, and 5 give the results obtained from the same three panel members as used in the Mark I analysis. The conditional probability of a match between the heuristic of a subject and that of the model is given in the last column.

In general, the probability of a match is small, and averages 0.16, approximately. Moreover, the observed matches resulted primarily from only two subjects, numbers 1 and 3. These subjects were also used in the Mark I experiment. Thus, if these subjects are excluded, the conditional probability of a match is essentially zero. The low probabilities resulted partly from the assignment of inappropriate priorities to the heuristics. The most frequently selected heuristic from the list of possible heuristics was the first heuristic listed in Table 1b. This heuristic ranked fifth in the pre-assigned priorities.

Table 15 shows the (unconditional) probability that a subject will state some heuristic contained in the list of possible heuristics. Here the average probability increases to approximately 0.50.

TABLE 14. COMPUTATION OF THE CONDITIONAL PROBABILITY THAT A SUBJECT'S HEURISTIC WILL MATCH THAT OBTAINED BY MARK II SIMULATION

Subtrajectory Number	Mark II Heuristic	Number of Subjects Having Same Heuristic as Mark II* (1)	(2)	(3)	Total T	Conditional Probability, T/(3) (14)
1	None	--	--	--	--	--
2	None	--	--	--	--	--
3	7,3	2	1	1	4	0.095
4	None	--	--	--	--	--
5	7,3,4	2	3	1	6	0.143
6	7,3,4,8	3	3	2	8	0.191
7	7,3,4	1	2	1	4	0.095
8	None	--	--	--	--	--
9	7,3,4	2	2	2	6	0.143
10	7,3	3	3	3	9	0.214
11	None	--	--	--	--	--
12	7,3	4	4	2	10	0.238

* Columns (1), (2), and (3) correspond to the three panel judges

TABLE 15. COMPUTATION OF THE CONDITIONAL PROBABILITY THAT A SUBJECT'S HEURISTIC WILL MATCH SOME HEURISTIC IN THE LIST OF POSSIBLE HEURISTICS

Subtrajectory Number	Number of Subjects Using Heuristic from List* (1)	(2)	(3)	Total T	Observed Frequency, T/(3) (14)
1	5	6	6	17	0.405
2	5	7	5	17	0.405
3	6	8	4	18	0.429
4	7	7	6	20	0.476
5	9	9	6	24	0.571
6	9	9	5	23	0.548
7	8	9	3	20	0.476
8	8	5	5	18	0.429
9	9	7	5	21	0.500
10	10	8	7	25	0.595
11	10	9	7	26	0.619
12	9	9	5	23	0.548

*Columns (1), (2), and (3) correspond to the three panel judges

Finally, it is noted that the penalty for missing the desired final position for these problems was found to be excessively large. Because of the difficulty of the control problem and because the penalty for missing the end-point was large, many of the subjects regarded the minimization of fuel as unimportant.

In summary, the results obtained in the analysis of the verbal statements for the Mark II experiment do not confirm the predicted heuristics produced by the model. However, it is conjectured that with a revised assignment of priorities, with smaller penalties for missing the terminal positions, and with more decisions per problem, the model may prove to be a reasonably good predictor of the verbal heuristics used by human controllers.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this report two mathematical models, Mark I and Mark II, for human decision-making in control systems are developed. Mark I simulates human decision-making in a first-order control problem, and Mark II simulates human decision-making in a second-order control system. In constructing the mathematical models, the following hypotheses are made. The human controller will search for "optimal" control policies, will generate heuristics based upon the observed data, and will use the heuristics as his control strategies. Experimental studies with human controllers were performed to test for these hypotheses of modeling. The experimental results obtained are quite encouraging, and the proposed models appear to be a reasonable approach to the problem.

The essence of the mathematical models lies in the sequential selection of control values in accord with four different control algorithms. At any given time, the control algorithm in operation depends upon the number of decisions made in the past, the number of decisions remaining, and the results of analysis of empirical data obtained entirely from meter readings. The control algorithms are further classified as follows: (1) Probing control algorithm and (2) incremental control algorithms. During the probing control, the models select control values, in succession, from a predetermined sequence. During incremental control, models select appropriate controls on the basis of changes in meter readings resulting from previous use of each of the controls. The incremental control is subdivided into three different modes: (a) terminal mode, (b) heuristic mode, and (c) gradient mode. In the terminal mode,for Mark I the control is so chosen that the difference between the required final velocity and the linear extrapolation of the current velocity is a minimum. For Mark II the control is chosen such that the difference between the required final position and the extrapolted value of position is a minimum. In the heuristic mode, the control choice is made to better establish, or maintain, an invariant relation among the meter readings found to occur when the incremental fuel consumption is minimal. The control is selected in order that the expected increment in the desired meter reading (or combination of meter readings) has the maximum magnitude and correct sign. In the gradient mode, the preceding control is chosen as the current control whenever the current incremental cost is less than the preceding incremental cost.

The primary objective of each model is to analyze data obtained during a control task so that control strategies, called heuristics, may be generated. These heuristics are derived on the basis of the invariant relations detected among meter readings, or combinations of meter readings, taken at regular intervals of time during the control task. If no invariant relations are detected, the model continues a search procedure and gathers more data. If invariant relations are detected, then these relations are used by the model in making the choice of appropriate control strategies. When the heuristics are generated by the model, it is predicted that human controllers would likewise find, and use, the same or equivalent heuristics, even though the search procedure and subsequent experience would be different for each human.

The heuristics evolved by the models are restricted to a list of possible heuristics, and consequently the human controller may evolve a heuristic not in the list. However, because the list of heuristics is "complete" in a certain sense, it has been demonstrated that if a human controller evolves a heuristic it will most likely match some heuristic in the list. To simplify the initial investigation, preassigned priorities are associated with each heuristic in the list. Problems involving selections among equivalent and conflicting heuristics are avoided in the initial study.

A theoretical basis of this research is derived form the Pi-Theorem of dimensional analysis. Because of the simplicity of the problems studied, only the invariance of the readings on single meters was involved. Thus, the dimensionless combinations of meter readings yielded by the Pi Theorem were not needed; and it follows that the theoretical basis of this research has not been fully investigated.

Based on the results obtained to date, it is not known whether the concern in this research with combinations of meter readings is really justified. Concern with the invariance of single meters may suffice. However, it is conceivable that highly trained and talented controllers may deal with combinations of meter readings. This may require the use of the theoretical basis in a more general form in order to evolve the heuristics of such controllers.

The heuristic mode of control was developed primarily as a procedure that would search for invariant relations among meter readings and use these relations, when found, as a basis for the selection of controls. In order that the model be capable of solving fixed endpoint control problems, it was found convenient to use three additional modes of control. It is clear that other computer procedures could have been used that also would permit the embedding of the heuristic mode of control. For the primary purpose of this research any such alternative procedure would have been acceptable. The model actually used represents, at best, a first-order attempt to implement the theoretical basis. It is conceivable that other procedures could produce the same heuristics, but would differ in the secondary performance measures, such as total fuel consumption. In this research very little consideration has been given to alternative procedures for embedding the search for invariance and the heuristic mode of control.

The basic data obtained from the experimental studies were verbal recommendations regarding the selection of controls. As expected, the subjects verbalized their recommendations in a variety of ways. As an example, the reference velocity, at which the incremental cost was zero, was verbalized as follows: minimum point, level, number, place, rest stop, and zero point. This example suggests that it was often difficult to extract the meaning of a statement made by a subject.

As suggested by the above remarks, the most difficult and subjective element in this proposed approach involves the association of the verbal statements of the subjects with the statements in the list of heuristics obtained from the theoretical framework. Although the agreement among the panel members was good, it would appear desirable to minimize this type of analysis. As one possible alternative, the list of heuristics, augmented with irrelevant but plausible heuristics, could be presented to the subject at the beginning of the experiments. After instructions on the meaning of each heuristic in the list, the subject could then be advised that after each problem he could send back his recommendations in his own words or choose any of the statements on the list.

In evaluating these results it must be re-emphasized that the high correlation between the fuel consumption of the models and the subject median fuel consumption was not expected. In fact, no great effort was made to try to construct the computer program to simulate the behavior of the human controller. Instead, the computer program was designed to use a simple search procedure in order to generate data. By combining this search procedure with the gradient mode of control, it was expected that minima in the incremental fuel costs would thereby be found. By reading (or interpolating) the meters at that time, the data required for invariance computations would be generated.

It is not asserted that the human controllers operate in the manner specified by the computer logic. Controllers may use widely different search procedures which are partly random and partly systematic. It is highly unlikely that all meters are read or interpolated as required by the model, and it is certainly not expected that any subject would use the coefficient of variation as his criterion for invariance. The basic assumption of the model is that human controllers will generate and analyze data and thereby evolve heuristics. Agreement between human controllers and models was hypothesized in the heuristics evolved, not in the procedures used to generate them. The high correlations reveal the possibility that the procedures may be similar. However, very detailed experimental work and analysis would be required to study such similarities.

Finally, it is noted that the performance of the subjects was measured in terms of the median fuel cost. This was done in order to eliminate the large effect of "outliers" on a measure of central tendency. This desirable feature is offset, perhaps, by the association of "learning curves" with the median of a group of subjects. It is clear that learning is basically defined for an individual, so that such a group learning curve may be misleading and not represent the learning curve of any individual in the group.

With the qualifications contained in the preceding evaluations, it is concluded that the proposed models offer a reasonable approach to the modeling of the verbal heuristics of human controllers for a first-order control system of the type investigated. A similar conclusion for a second-order control system is not justified by the results obtained to date. The proposed various modes of control in the mathematical models are applicable to other control problems than first-order and second-order control systems. The only differences will be in the form of the recursive formulas and transformational equations. The basic structure of the model can be used even when the control plant is only partially known.

In considering the future research that is suggested by this study, it is convenient to consider several different classes of problems:

(1) Make an application of the theory to a real-world control problem.

The problems considered in this study were initiated only for determining the feasibility of the method of approach. With feasibility demonstrated, it would appear desirable to attempt to make a realistic application. Ideally, such an application would have the following characteristics. A highly trained controller would be required to generate a minimum-fuel trajectory using training and simulation equipment. He would be restricted to the use of meters alone, and the cumulative fuel used to the current time would be displayed on one of the meters. At regular intervals he would be required to transmit to a hypothetical fellow astronaut, about to begin a similar control problem, any advice he could offer regarding the selection of controls. The statements should be taped and another astronaut or highly trained person should determine whether heuristics were evolved by the astronaut and whether these heuristics were predicted by an appropriately modified, Mark III, model.

(2) Obtain detailed descriptions of a given human controller.

For a given human controller, the parameters of the models could be adjusted to achieve the best possible fit to the output of a given human controller. Because there are several parameters in these models referring to the characteristics of the human (number of minima encountered before evolving a heuristic, the initiation of terminal control, threshold coefficients of variation, deviations tolerated between actual and devised magnitudes, priorities assigned to meters and combinations of meters, etc.), it is clear that rather specific sets of numerical values may be obtained for a given human controller. Once such evaluations are made, then predictive studies could be made in which the same subject would again be used after the models have predicted his trajectories and the heuristics used to generate them.

(3) Mark extensions of the current models to permit "learning".

The Mark I and Mark II models do not "learn". Although a "complete" list of heuristics is produced by the models, the selection of a heuristic from the list involves preassigned priorities. Ideally, the organization of models should be modified according to experience. The models also need to be modified to permit learning relative to the initiation of terminal control. The present models may initiate and suspend terminal operation several times during a trajectory. This suggests that the initiation of terminal control was early, and the model should learn to appropriately modify its criterion for terminal control.

(4) Extend the present models to permit several control variables.

The present models have a single control variable. These should be extended to several control variables having several levels for each.

(5) Extend the logic of the models to permit the use of several heuristics.

The present models use a single heuristic from the list of possible heuristics. Those models should be extended to permit the use of more than one heuristic at a time. Ideally, the models should "learn" which heuristics are equivalent, which conflict, and whether conflicting heuristics may be weighted or "blended" in some way.

(6) Make parametric studies of the existing models.

The existing models have many parameters which are the set of input variables to the computer program. The programming has been carried out to permit wide variations in the values of these parameters. The operation and behavior of the models would be more thoroughly discernable if a large number of computer runs were made with the parameters ranging over their permitted ranges. The following is a list of some of these parameter variations that would be particularly useful:

(a) Vary the number of meters and variables displayed to determine the effect on trajectory and the evaluation of heuristics by the models

(b) Vary the number of interpolations required before searching for invariant meter readings

(c) Vary the number of controls required to begin terminal operation

(d) Increase the number of levels for the control variable to approximate a continous control variable

(e) Vary the threshold coefficient of variation to determine the effect on evolution of heuristics by the model

(f) Use all possible nonsingular P-matrices to determine the effect on the evolution of heuristics involving more than one meter reading

(g) Vary the priorities assigned to the invariance of the individual meters and the combinations of meters

(h) Wherever the dimensional assignments are arbitrary (e.g. cost), vary the dimensions to find the effect of these assignments on the evolution of heuristics of the models.

In short, this research has laid some groundwork for the modeling of human decision-making in control problems. The design of a mathematical model which will incorporate sophisticated adaptive logic, associative memory and learning capability in executing the various modes of control--probing mode, gradient mode, terminal mode, and heuristic mode--appears to provide challenging problems for further research in mathematical modeling of human decision-making in control systems.

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APPENDIX A

A COMPLETE LISTING OF FORTRAN INSTRUCTIONS FOR THE MARK I AND MARK II MODELS

APPENDIX A

A Complete Listing of FORTRAN Instructions For the Mark I and Mark II Models

```
     DATE 7/05/66        AT  145422
PROGRAM MARK I
DIMENSION IDTEMP (19,26)
DIMENSION IEM (20)
DIMENSION IN (10)
DIMENSION IPLOT (45,55)
DIMENSION AMATRIX (20)
 DIMENSION SMV (8,20)
DIMENSION A (50)
DIMENSION C (50)
DIMENSION CFV(40)
DIMENSION CMEAN  (40)
DIMENSION CMV (20)
DIMENSION CPIMEAN (40)
DIMENSION CSGN (40)
DIMENSION DEC (50)
DIMENSION EMATRIX (19,26)
DIMENSION EPI (50)
DIMENSION EPLY (15)
DIMENSION ESTDG (15)
DIMENSION ESTDPI (15)
DIMENSION GOAL (51)
DIMENSION IMATRIX (19,19)
DIMENSION IMV (15,20)
DIMENSION INTMV (20)
DIMENSION IPI (15)
DIMENSION LCA (50)
DIMENSION LCB (50)
DIMENSION LOWL (50)
DIMENSION PERLY (15)
DIMENSION PMATRIX (7,7)
DIMENSION PMEAN (40)
DIMENSION PMV (20)
DIMENSION PPIMEAN (40)
DIMENSION PPMV (20)
DIMENSION PRIR (40)
DIMENSION PSGN (40)
DIMENSION PSTAR (50)
DIMENSION QMATRIX (19,7)
DIMENSION SCFV (40)
DIMENSION SPICB (8,40)
DIMENSION U (15)
DIMENSION UPL (50)
DIMENSION V (50)
DIMENSION VAR (40)
DIMENSION W (50)
REAL IMV
REAL INTMV
REAL IPI
REAL LCA
REAL LCB
REAL LOWL
REAL MEAS
REAL S
REAL SS
```

```
        DATE 7/05/66        AT  145422
      INTEGER C
      INTEGER CHGINDX
      INTEGER CMLIM
      INTEGER CSGN
      INTEGER D
      INTEGER DEC
      INTEGER H
      INTEGER L
      INTEGER LS
      INTEGER N
      INTEGER P
      INTEGER PMATRIX
      INTEGER PRIR
      INTEGER PSGN
      INTEGER QMATRIX
      INTEGER R
      INTEGER RDEC
      INTEGER T
      INTEGER TCRT
      INTEGER TERMC
      INTEGER DKS
      INTEGER NKS
      INTEGER INTERPL
      INTEGER K
      INTEGER J
      INTEGER EMATRIX
      INTEGER PMAX
      INTEGER NOPI
      INTEGER DS
      INTEGER KS
      EQUIVALENCE (EMATRIX,E)
      COMMON /CCOM/CHGINDX, CTABLE (8)
      COMMON/ECALC/PMATRIX,QMATRIX,EMATRIX
      DIMENSION IFMT1 (7)
      EQUIVALENCE ( IFMT1(3), IELENGTH)
      EQUIVALENCE ( IFMT1 (5), IEWIDTH)
      DIMENSION IDUM (26)
      DATA ( IFMT1 = 8H(* EMATR         ,
     $              8HIX*,/,/,          ,
     $              8H                  ,
     $              8H(1X,              ,
     $              8H                  ,
     $              8HI5,/),1H          ,
     $              8H )                )
C   IFMT1 IS A VARIABLE FORMAT STATEMENT USED TO PRINT OUT ≠EMATRIX≠.
C   I2000 IS A SWITCH USED TO KEEP FROM PRINTING THE TAPE STORED DATA,
C   MORE THAN ONE TIME AS THE PROGRAM RECYCLES THE 2000 BLOCK.
      I2000 = 0
C    MT ≠N, ARE NUMERICAL TAPE ASSIGNMENTS.
      MT 3 = 3
      MT 4 = 4
      MT 5 = 5
      REWIND MT3
      REWIND MT4
```

```
FTN 1.4            DATE 7/05/66        AT  145422
          REWIND MT5
      C   READ INPUT DATA AND STORE ON TAPE.
      1   READ 2, IN
          WRITE(5,2) IN
      2   FORMAT(10A8)
          IF ( EOF, 60) 3,1
      3   END FILE 5
          REWIND 5
      C   READ INPUT DATA FROM TAPE AND LIST.
      4   READ (5,2) IN
          IF ( EOF,5) 7,5
      5   PRINT 6,IN
      6   FORMAT(1X, 10A8)
          GO TO 4
      7   REWIND 5
      C PRINT LINE TO VOID ≠AUTO EJECT≠-- PROGRAM MAINTAINS A TALLY OF
      C OF LINES PRINTED.
          PRINT 9,
      9   FORMAT(*AAUTO EJECT RELEASE LINE.....................*,/,1H1)
      C  START TO READ INPUT DATA.  DATA MUST BE IN THE PROPER OEDER, AND
      C  THE PROPER NUMBER OF CARDS FOR EACH ARRAY.
      C  ALL TWO DIMENSION ARRAYS ARE READ IN BY ROWS.
          READ(5,90009), IXOUTPUT
  90009 FORMAT(9X,I1)
          READ ( 5,10) , M
       10 FORMAT(I5)
          READ ( 5,10) , PMAX
          READ ( 5,10) , CMLIM
          READ(5,11)      ,FINTERM
          READ ( 5,10) , TCRT
          READ ( 5,10) , R
          READ ( 5,10) , IPSIZE
          READ ( 5,10) , IQSIZE
          READ ( 5,10) , JQSIZE
          ITEMP = PMAX + 1
          READ ( 5,11) , (GOAL(I),I=1,ITEMP)
       11 FORMAT ( F5.2)
          READ ( 5,12) , (DEC(I),I=1,PMAX)
       12 FORMAT(I5)
          READ ( 5,11) , (LCA(I),I=1,PMAX)
          READ ( 5,11) , (LCB(I),I=1,PMAX)
          READ ( 5,11) , (V(I),I=1,PMAX)
          READ ( 5,11) , (W(I),I=1,PMAX)
          READ ( 5,11) , (A(I),I=1,PMAX)
          READ ( 5,10) , (C(I),I=1,PMAX)
          READ ( 5,11) , (PSTAR(I),I=1,PMAX)
          READ ( 5,11) , (LOWL(I),I=1,PMAX)
          READ ( 5,11) , (UPL(I),I=1,PMAX)
          READ ( 5,12) , ((PMATRIX(I,J),J=1,IPSIZE),I=1,IPSIZE)
       13 FORMAT (  F5.2)
          READ ( 5,11) , (EPI(I),I=1,PMAX)
          READ ( 5,12) , ((QMATRIX(I,J),J=1,JQSIZE),I=1,IQSIZE)
          J = 2 * M - R
          READ ( 5,10) , (PRIR(I),I=1,J)
```

```
FTN 1.4            DATE 7/05/66         AT  145422
          READ (5,10) ( IFM(I),I=1,M)
      C   READ IN THE SIZE OF THE A MATRIX--IASIZE IS THE ROWS--
      C      JASIZE IS THE COLS.
          READ ( 5,15) , IASIZE
          READ ( 5,15) , JASIZE
      15  FORMAT(I5)
          PRINT 16
      16  FORMAT(*1MARK I*,/,/,* DIMENSIONAL MATRIX*,/,4H*  M,2X,*L*,
         $2X,*T*,2X,*-*,/,12X,*0*,/,/)
      C   READ IN AMATRIX A ROW AT A TIME AND PRINT OUT.
          DO 20 ISUB =1,IASIZE
          READ ( 5,15) , (AMATRIX(JSUB),JSUB=1,JASIZE)
      20  PRINT 17, ( AMATRIX      (JSUB),JSUB=1,JASIZE)
      17  FORMAT(1X, 20I3)
          PRINT 18
      18  FORMAT(/,/,/,/)
      C  END READING OF INPUT DATA.
      C  100 BLOCK IS THE INITIALIZATION OF THE PROGRAM.
       100 P=1
          ICNT = 0
          IFRST = 0
          CHGINDX=0
          TEMP= 10.**100
          DO 120 ISUB=1,15
          DO 120 JSUB=1,20
       120 IMV(ISUB,JSUB)=TEMP
          C(P)=2
          DO 130 ISUB=1,40
       130 PSGN(ISUB)=PMEAN(ISUB)=PPIMEAN(ISUB)=0.
          ITEMP = M-R
          ENCODE ( 8, 1106, IELENGTH) ITEMP
      1106 FORMAT(I8)
          ENCODE ( 8,1106, IEWIDTH) M
          CALL FMAT      (R,M)
          PRINT 195
      141 FORMAT(* COMPUTATIONAL FORM*)
          WRITE ( 61, IFMT1) ((EMATRIX(ISUB,JSUB),JSUB=1,M),ISUB=1,ITEMP)
          PRINT 141
      195 FORMAT(/,/,* STANDARD FORM*)
          ITEMP = M - R
          DO 190 I=1, ITEMP
          DO 190 J=1,M
      190 IDTEMP(I,J) = EMATRIX(I,IFM(J))
          WRITE(61,IFMT1)((IDTEMP(I,J),J=1,M),I=1,ITEMP)
          DO 194 I = 1, ITEMP
          DO 194 J   1 , M
      194 EMATRIX(I,J) = IDTEMP(I,J)
          PRINT 196
      196 FORMAT(/,/,*  MATRIX*,/)
          DO 197 ISUB=1,IPSIZE
      197 PRINT 198, (PMATRIX(ISUB,JSUB),JSUB=1,IPSIZE)
      198 FORMAT(1X,7I4)
          PRINT 199
      199 FORMAT(/,/,* QMATRIX*,/)
```

```
FTN 1.4            DATE 7/05/66          AT  145422
          DO 20198 ISUB=1,IQSIZE
    20198 PRINT 198, (QMATRIX(ISUB,JSUB),JSUB=1,JQSIZE)
          PRINT 250,R
      250 FORMAT(*1 MANUAL CONTROL SIMULATION*,/,
         $1X,*MARKI*,/,/,/,
         $* CONTROL VALUE APPEARS ON METER NUMBER                 1*,/,/,
         $* NUMBER OF DECISIONS REMAINING-METER NUMBER            2*,/,/,
         $* VALUE OF STATE VARIABLE APPEARS ON METER NUMBER       3*,/,/,
         $* DISTANCE MEASURE FROM GOAL APPEARS ON METER NUMBER 4*,/, /,
         $* COST INCREMENT APPEARS ON METER NUMBER                5*,/,/,
         $* CUMULATIVE COST APPEARS ON METER NUMBER               6*,/,/,
         $* PARTITIONED DIMENSIONAL MATRIX OF RANK*,12X,I2)
          DO 150, I=1,45
          DO 150, J=1,55
    150   IPLOT (I,J) = 8H
          IPLTCNT = 2
    C     200 BLOCK IS THE BRANCH POINT TO START EACH NEW SUBTRAJECTORY.
      200 DO 210 ISUB=1,20
      210 CMV(ISUB)=PMV(ISUB)=PPMV(ISUB)=0.
          ITEMP = (( GOAL(P) - 440) / 10) + 1
          IPLOT(ITEMP,IPLTCNT) = 54H
          IPLTCNT = IPLTCNT + 1
          L = 1
          NOPI = DS =   LS = KS = 0
          DKS=NKS=TERMC=0
          J=H=K=0
          INTERPL = 0
          CHGINDX = 0
          IF ( P .NE. 1 ) CHGINDX = 2
          DO 220 ISUB=1,8
          DO 220 JSUB=1,20
      220 SMV(ISUB,JSUB)=0.
          CMV(2)= DEC(P)
          CMV(3)=GOAL(P)
          CMV(4)=GOAL(P+1)-GOAL(P)
          DO 230 ISUB=1,40
      230 CSGN(ISUB)=CMEAN(ISUB)=CPIMEAN (ISUB) = 0.
          DO 240 ISUB=1,15
      240 U(ISUB)=0.
          ITEMP = P
    99999 PRINT 251,ITEMP,GOAL(ITEMP),
         $GOAL(ITEMP+1),
         $                              DEC(ITEMP),A(ITEMP),CMLIM,
         $PSTAR(ITEMP),TCRT,LOWL(ITEMP),UPL(ITEMP),LCA(ITEMP),LCB(ITEMP),
         $V(ITEMP),W(ITEMP)
      251 FORMAT(1H1, 10X,*SUBTRAJECTORY  *,13X,I2,/,/,/,       /,
         $* INITIAL VALUE OF STATE VARIABLE*,17X,F15.2,/,/,
         $* DESIRED FINAL VALUE OF STATE VARIABLE*,11X,F15.2,/,/,
         $* DECISIONS AVAILABLE TO REACH FINAL VALUE*,21X,I2,/,/,
         $* REFERENCE LEVEL*,33X,F15.2,/,          /,
         $* MEMORY LIMIT*,49X,I2,/,        /,
         $* THRESHOLD COEFFICIENT OF VARIATION*,14X,F15.3,/,       /,
         $* NUMBER OF CONTROL LEVELS TO INITIATE TERMINAL CONTROL*,8X,I2,/,/
         $,* LOWER LIMIT ON FINAL VALUE OF STATE VARIABLE*,4X,F15.2,/,/,
```

```
FTN 1.4           DATE 7/05/66        AT  145422
          $* UPPER LIMIT ON FINAL VALUE OF STATE VARIABLE*,4X,F15.2,/, /,
          $* COEFFICIENT OF PREVIOUS STATE VALUE, A*,10X,F15.2,/,      /,
          $* COEFFICIENT OF CONTROL VALUE, B*,17X,F15.2,/,             /,
          $* COEFFICIENT OF COST INCREMENTS, V*,15X,F15.2,/,      /,
          $* COEFFICIENT OF FINAL MISS DISTANCE, W*,11X,F15.2)
           PRINT 413      ,P
    413    FORMAT (*1SUBTRAJECTORY*,I3,/,/,
          $8X,*CONTROL*,
          $8X,*REMAINING*,
          $6X,*CURRENT*,
          $8X,*VELOCITY*,
          $5X,*INCREMENTAL*,
          $4X,*CUMULATIVE*,    /,
          $8X,*CHOICE*,
          $9X,*DECISIONS*,
          $6X,*VELOCITY*,
          $7X,*ERROR*,
          $8X,*COST*,
          $11X,*COST*,  /,/,
          $       *         CHV (1)*,8X,
          $*CHV (2)*,8X,
          $*CHV (3)*,8X,
          $*CHV (4)*,8X,
          $*CHV (5)*,8X,
          $*CHV (6)*,8X,
          $/,/)
           ICNT = 3
C    300 BLOCK ESTABLISHES THE CONTROL VALUES.
  300 J=10-C(P)
  310 U(J)=J-10
          J=J+1
          IF(J .LE.10+C(P)) GO TO 310
C    *CSUB* IS A SUBROUTINE FOR PICKING CONTROL VALUES.
          CALL CSUB(J)
          GO TO 400
380       ICNT = 3
          PRINT 413    ,P
          GO TO 419
C    400 BLOCK IS THE TRANSFORMATION LAWS.
400       CONTINUE
          DO 410 ISUB=1,M
          PPMV(ISUB)=PMV(ISUB)
   410 PMV(ISUB)=CMV(ISUB)
          CMV(1)=U(J)
          CMV(2)=PMV(2)-1.
          CMV(3)=LCA(P)*PMV(3)+LCB(P)*CMV(1)
          CMV(4)=GOAL(P+1)-CMV(3)
          CMV(5)=((CMV(3)-A(P))**2)*V(P)
          CMV(6)=PMV(6)+CMV(5)
          IF ( IXOUTPUT .NE. 1 )        GO TO 90414
          PRINT 414, CHGINDX, INTERPL,CHLIM, TERMC, H, DS, LS, KS, NOPI, L
         $,ALPHA
414       FORMAT(*0CHGINDX = *,I5,/,
         $*0INTERPL = *,I5,/,
```

```
FTN 1.4          DATE 7/05/66        AT  145422
          $*0CMLIM = *,I5,/,
          $*0TERMC = * , I5,/,
          $*0M = *,I5,/,
          $*0DS =*,I5,/,
          $*0LS = *,I5,/,
          $*0KS = *,I5,/,
          $*0NOPI =*,I5,/,
          $*0L =*,I5,/,
          $*0ALPHA = *,     F15.3,1X)
    90414 CONTINUE
          ITEMP = (( CMV(3) - 440 ) / 10        ) + 1
          IPLOT(ITEMP,IPLTCNT) = 54H
          IPLTCNT = IPLTCNT + 1
          PRINT 415 ,(CMV(ISUB),ISUB=1,6)
    415   FORMAT(1H0,6F15.3)
          ICNT = ICNT + 2
          IF ( ICNT .GE. 54) GO TO      380
    419   DO 420 ISUB=1,M
      420 IMV(J,ISUB)=CMV(ISUB)-PMV(ISUB)
          IF ( IFRST .NE. 0 ) GO TO 490
          IF( CHGINDX.GT. 2*C(P)+1) GO TO 1500
    440   CALL CSUB(J)
          GO TO 400
    490   IFRST = IFRST + 1
          GO TO(440,                 1500) IFRST
C     500 BLOCK - INTERPOLATIONS AT ZERO COST INCREMENTS.
      500 IF(M.NE.1) GO TO 550
          IF(IMV(J,5).GT.0.) N     =N     -2
          N = N + 1
          D = D + 1
          MEAS        =  (N*1.) / (D*1.)
          IF(MEAS     .GE. .5) GO TO 1300
          L=L5+1
          GO TO 4000
      550 IF(CMV(5).NE.0.) GO TO 700
          K=1
      560 INTMV(K)=CMV(K)
          K=K+1
          IF(K.LE.M) GO TO 560
          INTERPL=INTERPL+1
          I=1
          DO 570 ISUB=1,M
      570 SMV(I,ISUB)=INTMV(ISUB)
          K=1
      580 I=CMLIM  + 1
    590   SMV(I,K) = SMV (I-1, K)
          I=I-1
          IF(I.GT.1) GO TO 590
          K=K+1
          IF(K.LE.M) GO TO 580
          IF(INTERPL.GE.CMLIM) GO TO 1000
C     600 BLOCK - CHOICE OF CONTROL BY MINIMIZING.
      600 ISUB=10-C(P)
          ITEMP= (C(P)*2)+ISUB
```

```
FTN 1.4              DATE 7/05/66          AT  145422
          XXMIN= 10.**101
          DO 620 JSUB=ISUB,ITEMP
          IF(IMV(JSUB,5)**2.GE.XXMIN) GO TO 620
          XXMIN=IMV(JSUB,5)**2
          J=JSUB
      620 CONTINUE
          GO TO 400
C       700 BLOCK - SEARCH ALGORITHM.
      700 IF(IMV(J,5).LT.0.) GO TO 400
          IF(CMV(1).EQ.PMV(1)) GO TO 750
          CALL CSUB(J)
          GO TO 400
    740   CALL CSUB(J)
          GO TO 400
      750 IF(PPMV(5).LT.PMV(5)) GO TO 740
          IF( PPMV(5)+CMV(5).EQ. 2* PMV(5)) GO TO 740
C       800 BLOCK - INTERPOLATION OF METER READINGS.
      800 ALPHA= (PPMV(5)-CMV(5))/(2.*(PPMV(5)-(2*PMV(5))+CMV(5)))
          K=1
          IF ( ALPHA.GE.0.) GO TO 850
      820 INTMV(K)=PMV(K)+(ALPHA*(PMV(K)-PPMV(K)))
          K=K+1
          IF(K .LE. M) GO TO 820
          GO TO 900
      850 INTMV(K)= PMV(K)+(ALPHA*(CMV(K)-PMV(K)))
          K=K+1
          IF(K .LE.M) GO TO 850
C       900 BLOCK - STORAGE OF METER READINGS.
    900   CONTINUE
          IF ( IXOUTPUT .NE. 1 ) GO TO 90900
          PRINT 88840, ( INTMV(ISUB),ISUB = 1,20)
    88840 FORMAT(* INTMV =  *,/,/,(1X,F20.3))
    90900 CONTINUE
          INTERPL = INTERPL +1
          I=1
          DO 910 KSUB=1,M
      910 SMV(I,KSUB)=INTMV(KSUB)
          K=1
      920 I=CMLIM  + 1
      930 SMV(I,K) = SMV(I-1,K)
          I=I-1
          IF(I .GT.1) GO TO 930
          K=K+1
          IF ( K .LE. M) GO TO 920
          IF (INTERPL .LT. CMLIM) 960, 1000
    960   CALL CSUB(J)
          GO TO 400
C       1000 BLOCK - COMPUTE COEFFICIENT OF VARIATION FOR METER READINGS.
     1000 NS = CMLIM
          S=SS=
          K=1
     1010 I=2
          SS = S =
     1020 S=S+SMV(I,K)
```

```
FTN 1.4            DATE 7/05/66        A1   145422
            SS=SS+ (SMV(I,K)**2)
            I=I+1
            IF(I .LE. CMLIM+1) GO TO 1020
            VAR(K)=(SS-((S*S)/(NS*1.)))/(NS*1.-1.)
            IF ( IXOUTPUT .NE. 1 ) GO TO 91020
            PRINT 89999, VAR(K),SS,S,NS,K
      89999 FORMAT(* VAR = *,F20.4,/,
           $* SS = *,F20.4,/,
           $* S = *,F20.4,/,
           $* NS = *,I5,/,
           $* K = *,I5,/,/)
      91020 CONTINUE
            CMEAN(K)= S/(NS*1.)
            CPIMEAN (K) = 1.
            SCFV(K)=VAR(K)/(CMEAN(K)**2)
            IF ( SCFV(K) .LT. 0. ) SCFV(K) = - SCFV(K)
            CFV(K) = SQRTF(SCFV(K))
            IF( CFV(K).GE. PSTAR(P)) GO TO 1100
       1050 NOPI=NOPI+1
            CSGN(K)=1
            WRITE (MT1) K, CMEAN(K)
      C    1100 + 1200 BLOCKS - COMPUTE COEFFICIENT OF VARIATION FOR
      C     COMBINATIONS OF METER READINGS.
       1100 K=K+1
            IF(K .LE.M) GO TO 1010
            K=M+1
            KSUB = 1
       1110 I=CMLIM  + 1
       1120 TEMP=1.
            DO 1130 ISUB=1,M
            IF ( SMV(I,ISUB) .EQ. 0.) GO TO 1130
            TEMP=TEMP*(SMV(I,ISUB)**EMATRIX(KSUB,ISUB))
      1130  CONTINUE
            SPICB(I,K)=TEMP
            I=I-1
            IF(I.GT.1) GO TO 1120
            KSUB = KSUB + 1
       1150 K=K+1
            IF(K.LE.2*M-R) GO TO 1110
            IF ( IXOUTPUT .NE. 1 ) GO TO 91160
            PRINT 88920, ((SPICB(ISUB,JSUB),JSUB=1,10),ISUB=1,8)
      88920 FORMAT(*1SPICB = *,/,
           $(10(1X,F12.6)))
      91160 CONTINUE
       1160 NS=CMLIM
            SS=S=0
            K=M+1
       1170 I=2
            S = SS = 0
       1180 S=S+SPICB(I,K)
            SS=SS+ (SPICB(I,K)**2)
            I=I+1
            IF(I .LE. CMLIM+1) GO TO 1180
       1200 VAR(K)=(SS-((S*S)/(NS*1.)))/((NS*1.)-1.)
```

```
FTN 1.4              DATE 7/05/66          AT  145422
            CPIMEAN(K) = S/(NS*1.)
            SCFV(K)=VAR(K)/(CPIMEAN(K)**2)
            IF ( SCFV(K) .LT. 0. ) SCFV(K) = - SCFV(K)
            CFV(K)=SQRTF(SCFV(K))
            IF ( IXOUTPUT .NE. 1 ) GOTO 91200
            PRINT 89999, VAR(K),SS,S,NS,K
      91200 CONTINUE
            IF(CFV(K).GE.PSTAR(P)) GO TO 1220
            NOPI=NOPI+1
            CSGN(K)=1
            ITEMP = K - M
            DO 1218 JSUB =1,M
      121.  IDTEMP(ITEMP,JSUB) = EMATRIX(ITEMP,     JSUB )
            WRITE (MT.) CPIMEAN(K), (IDTEMP(ITEMP,JSUB),JSUB=1,M)
       1220 K=K+1
            IF(K .LE. 2*M-R) GO TO 1170
            IF ( NOPI.LE. 0) GO TO 1250
            I=1
            GO TO 4000
      1250  CALL CSUB(J)
            GO TO 400
      C     1300 + 1400 BLOCKS - CHOICE OF CONTROL ON HEURISTIC.
      1300  CONTINUE
            IF ( IXOUTPUT .NE. 1 ) GO TO 91300
                                    PRINT 88810,CPI ,CHV(KS),CHEAN(KS),KS
      88810 FORMAT(     /,/,* CPI = *, F 20.3,
           $*  CHV(KS) = *,F10.3,*  CHEAN(KS) = *,F10.3,*  KS = *,I2)
            PRINT 88820,(IPI(JSUB),JSUB=1,15)
      88820 FORMAT(/,/,/, * IPI = *,/,/,  (1X,F20.3))
            PRINT 88860, CPIMEAN(KS),EPI(KS),KS
      88860 FORMAT(/,/,* CPIMEAN(KS) = *,E20.3,*  EPI(KS) = *,E20.3,
           $*   KS = *,I2)
      91300 CONTINUE
            IF(CPIMEAN(KS)-EPI(P).LE. CPI     .AND.
           $CPIMEAN(KS)+EPI(P ).GE. CPI     ) GO TO 1400
            J=16-C(P)
       1320 IF(IPI(J).NE.0.) GO TO 1330
      1325  PERLY(J)= -10.**100
            GO TO 135
       1330 ESTDPI(J)=(CPIMEAN(KS)-CPI     )/IPI(J)
            IF(ESTDPI(J) .LE. 0.) GO TO 1325
            PERLY(J)=CMV(P)-ESTDPI(J)
            IF (PERLY(J).LE.0.) PERLY(J)=-10.**100
       1350 J=J+1
            IF(J .LE. 15+C(P)) GO TO 1320
            ISUB =16-C(P)
            ITEMP=(C(P)*  )+ISUB
            XXMIN=-10.**101
            D  137  JSUB=ISUB,ITEM
            IF(PERLY(JSUB) .LT. XXMIN) GO TO 1370
            XXMIN=PERLY(JSUB)
            J=JSU
       1370 CONTINUE
            GO TO 400
```

```
FTN 1.4          DATE 7/05/66        AT  145422
 1400 ISUB= 10-C(P)
      ITEMP=(C(P)*2)+ISUB
      XXMIN = 10.**101
      DO 1420 JSUB=ISUB,ITEMP
      IF ( IPI(JSUB) **2 .GE. XXMIN ) GO TO 1420
      XXMIN=IPI(JSUB) ** 2
      J=JSUB
 1420 CONTINUE
      GO TO 400
C   1500 BLOCK - DETERMINS IF TERMINAL CONTROL SHOULD BEGIN.
 1500 IF( CMV(2).GT.0.) GO TO 1510
      IF(TERMC.GT.0) GO TO 1800
      PRINT 1505, P
 1505 FORMAT(* SUBTRAJECTORY *,I2,* ENDED USING SEARCH PROCEDURE.*,/,/)
      GO TO 1800
 1510 IF(GOAL(P+1)-LOWL(P).LE.CMV(3).AND.
     $GOAL(P+1)+UPL(P).GE.CMV(3)) GO TO 1890
      T=0
      JSUB = 10 - C(P)
 1530 IF ( IMV(JSUB,4) .EQ. 0. ) GO TO 1540
      ESTDG(JSUB) = - CMV(4) / IMV (JSUB,4)
      IF ( ESTDG(JSUB ) .GT. 0. ) GO TO 1545
 1540 ERLY (JSUB) = -10.**100
      GO TO 1560
 1545 ERLY(JSUB) = CMV(2) - ESTDG(JSUB)
 1550 IF ( ERLY(JSUB) .LT. 0. ) GO TO 1540
      T = T + 1
 1560 JSUB = JSUB + 1
      IF ( JSUB .LE. 10 + C(P)) GO TO 1530
      IF(T .LE.TCRT) GO TO 1570
      TERMC = 0
      IF (    CMV(2) .LE. FINTERM ) GO TO 1570
      CPI = CMV(KS) / CMEAN(KS)
      GO TO 500
 1570 TERMC=TERMC+1
      IF(T.GT.0) 1600,1700
C   1600 + 1700 BLOCK - CHOICE OF CONTROL FOR TERMINAL OPERATION.
 1600 ISUB=10-C(P)
      ITEMP=(C(P)*2)+ISUB
      XXMIN= 10.**101
      DO 1620 JSUB=ISUB,ITEMP
      IF(ERLY(JSUB)**2.GE.XXMIN) GO TO 1620
      XXMIN=ERLY(JSUB)**2
      J=JSUB
 1620 CONTINUE
      GO TO 400
 1700 ISUB=10-C(P)
      ITEMP=(2*C(P))+ISUB
      XXMIN=10.**101
      DO 1720 JSUB=ISUB,ITEMP
      IF(ESTDG(JSUB)**2.GE.XXMIN) GO TO 1720
      XXMIN=ESTDG(JSUB)**2
      J=JSUB
 1720 CONTINUE
```

```
FTN 1.4          DATE 7/05/66        AT  145422
      GO TO 400
C     1800 BLOCK - DETERMINE PENALTY AND TOTAL COST.
1800  ALOWL = GOAL(P+1) - LOWL(P)
      AUPL = GOAL(P+1)     +UPL(P)
      IF(ALOWL .LE.CMV(3).AND.
     $AUPL.GE. CMV(3)) GO TO 1810
      IF(CMV(3).LT.ALOWL) GO TO 1805
      TCST=        W(P)*((CMV(3)-AUPL)**2)
      PRINT 1804, TCST
1804  FORMAT(*0PENALTY IS*,F20.3)
      TCST = TCST + CMV(6)
      PRINT 1806, TCST
1806  FORMAT(* TOTAL COST IS *,F20.2)
      GO TO 1820
 1805 TCST=        W(P)*((ALOWL-CMV(3))**2)
      PRINT 1804, TCST
      TCST = TCST + CMV(6)
      PRINT 1806, TCST
      GO TO 1820
1810  TCRT = TCRT - 1
      IF ( TCRT .LT. 1) TCRT = 1
      TCST = CMV(6)
      PRINT 1807 , TCST
1807  FORMAT(* PENALTY IS 0*,/,* TOTAL COST IS *,F20.2)
 1820 K=0
      ISW=1
      GO TO 2000
1890  IF (TERMC.GT.0) GO TO 1900
      IF ( IMV(J,5) .LT. 0.) GO TO 400
      CALL CSUB(J)
      GO TO 400
C     1900 BLOCK - CHOICE OF CONTROL FOR TERMINAL OPERATION.
 1900 ISUB=10-C(P)
      ITEMP=(C(P)*2)+ISUB
      XXMIN=10.**101
      DO 1920 JSUB=ISUB,ITEMP
      IF(IMV(JSUB,4)**2.GE.XXMIN) GO TO 1920
      XXMIN=IMV(JSUB,4)**2
      J=JSUB
 1920 CONTINUE
      GO TO 400
C     2000 BLOCK - DETERMINATION OF HEURISTICS AND CONFIDENCE MEASURES
C     AND MEMORY LIMIT FOR NEXT SUBTRAJECTORY.
2000  CONTINUE
      IF ( I2000 .EQ. 1 ) GO TO 2009
      I2000 = 1
      END FILE MT3
      END FILE MT4
      REWIND MT 3
      REWIND MT 4
      IF ( INTEPPL .LT. CHLIM ) GO TO 2009
      ICNT = 1
      PRINT 2001,P
2001  FORMAT(*1METERS READING CONSTANT FOR SUBTRAJECTORY*,I3,/,/)
```

```
FTN 1.4          DATE 7/05/66          AT  145422
 2002  READ (MT3) IDUM1, IDUM2
       IF ( EOF, MT3) 2005,  2004
 2004  PRINT 1061, IDUM1, IDUM2
       ICNT = ICNT + 1
       IF ( ICNT .GE. 54 ) GO TO 2017
 1061  FORMAT(*0WHEN COST INCREMENTS ARE MINIMAL, METER *,
      $ I2,* READS *,F17.2)
       GO TO 2002
 2005  READ (MT4) IDUM1, (IDUM(ISUB),ISUB=1, M)
       IF ( EOF,MT4) 2008,  2006
 2006  PRINT 1205, IDUM1, ( IDUM(ISUB),ISUB=1, M)
       ICNT = ICNT + 1
       IF ( ICNT .GE. 54 ) GO TO 2019
 1205  FORMAT(*0WHEN COST INCREMENTS ARE MINIMAL, THE FOLLOWING COMBINAT*
      $,*ION OF METER READINGS EQUALS *,F17.2
      $,/,26I5)
       GO TO 2005
 2008  REWIND MT3
       REWIND MT4
 2009  JSW = 0
       K=K+1
       IF(K.GT.2*M-P) GO TO 2050
       IF(PSGN(K)*CSGN(K).NE.1.) GO TO 2000
       TEMP = ( 1. + DS ) / ( 2. + DS )
       IF(K.EQ.K5) GO TO 2018
  2010 IF(ISW.LE.2) ISW=2
       IF ( L .LE. M .AND. PMEAN(K).EQ. CMEAN(K)) GO TO 2020
       IF ( K .GT. M .AND. PPIMEAN(K) .EQ. CPIMEAN(K)) GO TO 2020
       PRINT 2015,K
  2015 FORMAT(*0DIMENSIONLESS PARAMETER NUMBER *,I2,
      $* IS INVARIANT WITHIN BUT NOT BETWEEN SUBTRAJECTORIES *)
       GO TO 2000
 2017  PRINT 2001
       ICNT = 3
       GO TO 2002
 2019  PRINT 2001
       ICNT = 3
       GO TO 2005
 2018  PRINT 2005, K, TEMP
  2005 FORMAT(*0CONFIDENCE MEASURE OF HEURISTIC BASED ON *,
      $*DIMENSIONLESS PARAMETER NUMBER *,I2,* IS EQUAL *,
      $*TO *,F20.4)
       JSW = 1
       GO TO 2010
  2020 IF ( JSW .EQ. 1 ) ISW = 3
       PRINT 2025,K
  2025 FORMAT(*0DIMENSIONLESS NUMBER *,I2,* IS INVARIANT *,
      $*WITHIN AND BETWEEN SUBTRAJECTORIES *)
       GO TO 2000
  2050 JSUB=2*M-R
       DO 2060 ISUB=1,40
       PPIMEAN(ISUB)=CPIMEAN(ISUB)
       PMEAN (ISUB) = CMEAN(ISUB)
  2060 PSGN(ISUB) = CSGN(ISUB)
```

```
FTN 1.4              DATE 7/05/66          AT   145422
         IF (ISW .EQ.3) CMLIM=CMLIM-2
         IF(ISW.EQ.2) CMLIM=CMLIM-1
         IF(CMLIM.LT.1)CMLIM=1
         DO 2070 ISUB = 1, 45
         IF ( ISUB .EQ. ((GOAL(P+1) - 440) / 10) + 1 ) GO TO 2070
         IF ( IPLOT(ISUB,DEC(P) + 2).EQ. 8H         )
        $IPLOT(ISUB,DEC(P) +2 ) = 0
  2070   CONTINUE
         ITEMP = (( GOAL(P+1) - 440) / 10) + 1
         IPLOT(ITEMP,DEC(P)+2) = 678
         IF ( CMV(4) .EQ. 0.) IPLOT(ITEMP,DEC(P)+2 ) = 548
         ITEMP = 450
         DO 2075 ISUB=2,45
         IPLOT(ISUB,1) = ITEMP
  2075   ITEMP = ITEMP + 10
         ITEMP = DEC(P)
         DO 2080 ISUB = 2, 55
         IPLOT(1,ISUB) = ITEMP
         ITEMP = ITEMP - 1
         IF ( ITEMP .LT. 0 ) GO TO 2085
  2080   CONTINUE
  2085   ITEMP = DEC(P)     +2
         JTEMP =  46
         PRINT 2087,P
  2087   FORMAT(*1VELOCITY AS A FUNCTION OF REMAINING DECISIONS*,
        $* FOR SUBTRAJECTORY*,I3,/,/)
         DO 2090 ISUB = 2 , 45
         JTEMP = JTEMP - 1
  2090   PRINT 2095, (IPLOT(JTEMP,JSUB),JSUB=1,ITEMP)
  2095   FORMAT(1X,I3, 55(2X,R1))
         PRINT 2086, ( IPLOT(1,ISUB),ISUB=2,ITEMP)
  2086   FORMAT(1X,3X, 55(1X,I2))
         IPLTCNT = 2
         DO 2096 ISUB = 1,45
         DO 2096 JSUB =1,55
  2096   IPLOT (ISUB,JSUB) = 8H
         P=P+1
         I2000 = 0
         IF ( P .EQ. 8 ) GOAL(P) =  570.0
         IFRST = 1
         REWIND MT3
         REWIND MT4
         IF(P.LE.PMAX) GO TO 200
         PRINT 3000
  3000   FORMAT(*1END OF TRAJ.*)
         CALL EXIT
  4000   K = 1
  4010   IF ( PRIM(K) .EQ. L ) GO TO 4015
         K = K + 1
         GO TO 4010
  4015   IF ( CSGN(K) .EQ. 1 ) GO TO 4017
  4016   L = L + 1
         IF ( L .LE. 2*M-R) GO TO 4000
         M = 0
```

```
FTN 1.4          DATE 7/05/66        AT  145422
           CALL C SUB (J)
           GO TO 400
     4017  M = 1
           DS = CMV (2)
           LS = L
           KS = K
           N = 1
           D = 2
           MEAS = 0.5
           IF ( KS .GE. M + 1 ) GOTO 4045
           IF ( CMEAN (KS) .EQ. 0. ) GO TO 4016
           CPI = CMV(KS) / CMEAN(KS)
           J = 10 - C(P)
     4030  IPI(J) = IMV(J,KS ) / CMEAN(KS)
           J = J + 1
           IF ( J .GT. C(P) + 10) 1300, 4030
     4045  TEMP = 1.
           KSUB = KS - M
           DO 4050 ISUB = 1, M
           IF ( CMV(ISUB) .EQ. 0.) GO TO 4050
           TEMP = TEMP * (CMV(ISUB)** EMATRIX(KSUB,ISUB) )
     4050  CONTINUE
           CPI = TEMP
           J = 10 - C(P)
           TEMP = 0.
     4055  DO 4060 ISUB = 1,M
     4060  TEMP = TEMP +((EMATRIX(KSUB,ISUB)*IMV(J,ISUB)) / CMV(ISUB))
           IPI(J)     = CPI * TEMP
           J = J + 1
           IF ( J .GT. 10 + C(P)) GO TO 1300
           GO TO 4055
           END
```

```
FTN 1.4          DATE 7/05/66        AT  145422
      SUBROUTINE CSUB(J)
      COMMON/CCOM/CHGINDX,CTABLE(8)
      DATA ( CTABLE=0,0,1,-1,2,-2,1,-1),(CHGINDX=0)          $
      TYPE INTEGER CHGINDX,CTABLE
      J=CHGINDX-(CHGINDX/8)*8+1
      J=CTABLE(J)    +10                                     $
      CHGINDX=CHGINDX+1
      RETURN
      END
```

```
FTN 1.4          DATE 7/05/66        AT  145422
            SUBROUTINE EHAT(R,M)
      C THIS PROGRAM HAS BEEN CHECKED OUT AND IS ≠OK≠  602066
      C     SUBROUTINE EHAT COMPUTES EHATRIX= (-QPINV,K*I) WHERE Q+P ARE
      C     FURNISHED THRU LABELED COMMON. DIMENSIONS ARE P(R,R), Q(M-R,R)
      C     AND I (M-R,M-R). PINV IS COMPUTED FROM P AND HENCE P MUST BE
      C     NON-SINGULAR. K IS A POSITIVE INTEGER CONSTANT WHICH IS THE
      C     SMALLEST INTEGER WHICH WILL ALLOW PINV TO HAVE ALL INTEGER
      C     ENTRIES
            COMMON/ECALC/P(7,7),Q(19,7),E(19,26)
            DIMENSION ITEMP(7,14),IPRIME(5)
            DATA (IPRIME=2,3,5,7,11)
            INTEGER P,Q,E,R
            DO 1 J=1,R
            DO 1 I=1,R
            ITEMP(I ,J)=P(I,J)
          1 ITEMP(I,J+R)=0
            DO 2 I=1,R
          2 ITEMP(I,I+R)=1
            IR=R+R
      C     CONSTRUCT ITEMP = (P,I)
            DO 3 IP=1,R
            IPIV=ITEMP(IP,IP)
      C     REDUCE P TO DIAGONAL MATRIX BY INTEGER ROW TRANSFORMATIONS.
            IF(IPIV) GO TO 20
            DO 22 I=IP,R
            IF(ITEMP(I,IP)) GO TO 21
         22 CONTINUE
            GO TO 19
         21 DO 23 J=IP,IR
         23 ITEMP(IP,J)=ITEMP(IP,J)+ITEMP(I,J)
            IPIV=ITEMP(IP,IP)
         20 DO 3 I=1,R
            IF(I.EQ.IP)GO TO 3
            IPPIV=ITEMP(I,IP)
            DO 5 J=1,IR
          5 ITEMP(I,J)=ITEMP(I,J)*IPIV-IPPIV*ITEMP(IP,J)
          3 CONTINUE
      C     COMPUTE LEAST COMMON POSITIVE MULTIPLE OF DIAGONAL ELEMENTS
            IPROD=1
            DO 4 I=1,R
            IPIV=ITEMP(I,I)
            IF((IPROD/IPIV)*IPIV.EQ.IPROD) GO TO 4
            IPROD=IPROD*IPIV
          4 CONTINUE
            IF(IPROD.LT.0) IPROD=-IPROD
            IRP1=R+1
      C     MULTIPLY PINV BY ROW BY LCM.
            DO 6 I=1,R
            MULT=IPROD/ITEMP(I,I)
            ITEMP(I,I)=IPROD
            DO 6 J=IRP1,IR
          6 ITEMP(I,J)=ITEMP(I,J)*MULT
      C     DETERMINE IF PIS SINGULAR.
            IF(IPIV.EQ.0)GO TO 19
```

```
FTN 1.4          DATE 7/05/66        AT  145422
      IP=1
C     REMOVE FACTORS OF PINV AND K.
    9 IPIV=IPRIME(IP)
   13 IF((ITEMP(1,1)/IPIV)*IPIV.EQ.ITEMP(1,1)) GO TO 7
   14 IP=IP+1
      IF(IP.GT.S) 8,9
    7 DO 10 J=IRP1,IR
      DO 10 I=1,R
      IF((ITEMP(I,J)/IPIV)*IPIV.EQ.ITEMP(I,J)) 10,11
   10 CONTINUE
      ITEMP(1,1)=ITEMP(1,1)/IPIV
      DO 12 J=IRP1,IR
      DO 12 I=1,R
   12 ITEMP(I,J)= ITEMP(I,J)/IPIV
      GO TO 13
   11 IF(IABS(ITEMP(I,J)).LT.IPIV)8,14
    8 IRP =M-R
C     FORM -Q*PINV
      DO 15 I=1,IRP
      DO 15 J=1,R
      IPIV=0
      DO 16 K=1,R
   16 IPIV=IPIV-Q(I,K)*ITEMP(K,J+R)
   15 E(I,J)=IPIV
C     ADD K*I TO EMATRIX
      DO 17 I=1,IRP
      DO 17 J=IRP1,M
   17 E(I,J)=0
      DO 18 IP=1,IRP
   18 E(IP, R   +IP)=ITEMP(1,1)
C     RETURN WITH EMATRIX = (-Q*PINV, K*I)
      RETURN
C     EXIT IF P IS SINGULAR.
   19 PRINT 102,R,((ITEMP(I,J),I=1,R),J=1,IR)
  102 FORMAT(27H ***P-MATRIX IS SINGULAR,R=,I2/(7I10))
      CALL EXIT
      END
```

```
FTN 1.4          DATE 7/05/66        AT  150515
      DIMENSION VAR (40)
      DIMENSION W (50)
      REAL IMV
      REAL INTMV
      REAL IPI
      REAL LOWL
      REAL MEAS
      REAL S
      REAL SS
      INTEGER C
      INTEGER CHGINDX
      INTEGER CMLIM
      INTEGER CSGN
      INTEGER D
      INTEGER DEC
      INTEGER H
      INTEGER L
      INTEGER LS
      INTEGER N
      INTEGER P
      INTEGER PMATRIX
      INTEGER PRIP
      INTEGER PSGN
      INTEGER QMATRIX
      INTEGER R
      INTEGER RDEC
      INTEGER T
      INTEGER TCRT
      INTEGER TERMC
      INTEGER DKS
      INTEGER NKS
      INTEGER INTERPL
      INTEGER K
      INTEGER J
      INTEGER EMATRIX
      INTEGER PMAX
      INTEGER NOPI
      INTEGER DS
      INTEGER KS
      EQUIVALENCE (EMATRIX,E)
      COMMON /CCOM/CHGINDX, CTABLE (8)
      COMMON/ECALC/PMATRIX,QMATRIX,EMATRIX
      DIMENSION IFMT1 (7)
      EQUIVALENCE ( IFMT1(3), IELENGTH)
      EQUIVALENCE ( IFMT1 (5), IEWIDTH)
      DIMENSION IDUM (26)
      DATA ( IFMT1 = 8H(* EMATR             ,
     $                    8HIX*,/,/,          ,
     1                    4H                 ,
     1                    8H(1X,             ,
     5                    4H                 ,
     1                    8HI5,/),1H         ,
     1                    8H )               )
C  IFMT1 IS A VARIABLE FORMAT STATEMENT USED TO PRINT OUT *EMATRIX*.
```

```
FTN 1.4             DATE 7/05/66         AT  150515
      C  I2000 IS A SWITCH USED TO KEEP FROM PRINTING THE TAPE STORED DATA,
      C  MORE THAN ONE TIME AS THE PROGRAM RECYCLES THE 2000 BLOCK.
            I2000 = 0
      C   MT #N# ARE NUMERICAL TAPE ASSIGNMENTS.
            MT 3 = 3
            MT 4 = 4
            MT 5 = 5
            REWIND MT3
            REWIND MT4
            REWIND MT5
      C   READ INPUT DATA AND STORE ON TAPE.
      1     READ 2, IN
            WRITE(5,2) IN
      2     FORMAT(10A8)
            IF ( EOF, 60) 3,1
      3     END FILE 5
            REWIND 5
      C   READ INPUT DATA FROM TAPE AND LIST.
      4     READ (5,2) IN
            IF ( EOF,5) 7,5
      5     PRINT 6,IN
      6     FORMAT(1X, 10A8)
            GO TO 4
      7     REWIND 5
      C  PRINT LINE TO VOID #AUTO EJECT#-- PROGRAM MAINTAINS A TALLY OF
      C  OF LINES PRINTED.
            PRINT 9,
      9     FORMAT(*AAUTO EJECT RELEASE LINE.......................*,/,1H1)
      C   START TO READ INPUT DATA,  DATA MUST BE IN THE PROPER ORDER, AND
      C   THE PROPER NUMBER OF CARDS FOR EACH ARRAY.
      C   ALL TWO DIMENSION ARRAYS ARE READ IN BY ROWS.
            READ(5,90009), IXOUTPUT
      90009 FORMAT(9X,I1)
            READ ( 5,10) , M
         10 FORMAT(I5)
            READ ( 5,10) , PMAX
            READ ( 5,10) , CNLIM
            READ ( 5,11) , FINTERM
            READ ( 5,10) , TCNT
            READ ( 5,10) , R
            READ ( 5,10) , IPSIZE
            READ ( 5,10) , IQSIZE
            READ ( 5,10) , JQSIZE
            ITEMP = PMAX + 1
            READ ( 5,11) , (GOAL(I),I=1,ITEMP)
         11 FORMAT ( F5.2)
            READ ( 5,12) , (DEC(I),I=1,PMAX)
         12 FORMAT(I5)
            READ ( 5,11) , (CST(I),I=1,PMAX)
            READ ( 5,11) , (ETA(I),I=1,PMAX)
            READ ( 5,11) , (V(I),I=1,PMAX)
            READ ( 5,11) , (W(I),I=1,PMAX)
            READ ( 5,11) , (A(I),I=1,PMAX)
            READ ( 5,10) , (C(I),I=1,PMAX)
```

```
FTN 1.4          DATE 7/05/66        AT  150515
          PROGRAM MARK II
C         STATEMENT NUMBERS FOR MARK II START AT 7000
          DIMENSION ETA (50)
          DIMENSION VEL (50)
          DIMENSION ACCEL (50)
          DIMENSION CSI (50)
          INTEGER BO
          INTEGER CV
          REAL EFFY
          REAL ETA
          REAL CSI
          REAL FACTOR
          REAL HCMV
          REAL VEL
          DIMENSION IDTEMP (19,26)
          DIMENSION IEM (20)
          DIMENSION IN (10)
          DIMENSION IPLOT(135,55)
          DIMENSION AMATRIX (20)
           DIMENSION SMV (8,20)
          DIMENSION A (50)
          DIMENSION C (50)
          DIMENSION CFV(40)
          DIMENSION CMEAN  (40)
          DIMENSION CMV (20)
          DIMENSION CPIMEAN (40)
          DIMENSION CSGN (40)
          DIMENSION DEC (50)
          DIMENSION FMATRIX (19,26)
          DIMENSION EPI (50)
          DIMENSION EELY (15)
          DIMENSION ESTDG (15)
          DIMENSION ESTDPI (15)
          DIMENSION GOAL (51)
          DIMENSION IMATRIX (19,19)
          DIMENSION IMV (15,20)
          DIMENSION INTMV (20)
          DIMENSION IPI (15)
          DIMENSION LOWL (50)
          DIMENSION PERLY (15)
          DIMENSION PMATRIX (7,7)
          DIMENSION PMEAN (40)
          DIMENSION PMV (20)
          DIMENSION PPIMEAN (40)
          DIMENSION PPMV (20)
          DIMENSION PPIR (40)
          DIMENSION PSGN (40)
          DIMENSION PSTAR (50)
          DIMENSION QMATRIX (19,7)
          DIMENSION SCFV (40)
          DIMENSION SPICO (8,40)
          DIMENSION U (15)
          DIMENSION UPL (50)
          DIMENSION V (50)
```

```
FTN 1.4            DATE 7/05/66          AT  150515
          READ ( 5,11) , (PSTAR(I),I=1,PMAX)
          READ ( 5,11) , (LOWL(I),I=1,PMAX)
          READ ( 5,11) , (UPL(I),I=1,PMAX)
          READ ( 5,11),(VEL(I),I=1,PMAX)
          READ (5,11),(ACCEL(I),I=1,PMAX)
          READ ( 5,12) , ((PMATRIX(I,J),J=1,IPSIZE),I=1,IPSIZE)
       13 FORMAT (  F5.2)
          READ ( 5,11) , (EPI(I),I=1,PMAX)
          READ ( 5,12) , ((OMATRIX(I,J),J=1,JASIZE),I=1,IOSIZE)
          J = 2 * M - R
          READ ( 5,10) , (PRIR(I),I=1,J)
          READ (5,16) ( IEM(I),I=1,M)
C         READ IN THE SIZE OF THE A MATRIX--IASIZE IS THE ROWS--
C            JASIZE IS THE COLS.
          READ ( 5,15) , IASIZE
          READ ( 5,15) , JASIZE
15        FORMAT(I5)
          PRINT 16
16        FORMAT(*1MARK II*,/,/,* DIMENSIONAL MATRIX*,/,4H*  M,2X,*L*,
         $2X,*I*,2X,*-*,/,12X,*0*,/,/)
C         READ IN AMATRIX A ROW AT A TIME AND PRINT OUT.
          DO 20 ISUB =1,IASIZE
          READ ( 5,15) , (AMATRIX(JSUB),JSUB=1,JASIZE)
20        PRINT 17, ( AMATRIX      (JSUB),JSUB=1,JASIZE)
17        FORMAT(1X, 20I3)
          PRINT 18
18        FORMAT(/,/,/,/)
C       END READING OF INPUT DATA.
C       100 BLOCK IS THE INITIALIZATION OF THE PROGRAM.
      100 P=1
          ICNT = 0
          IFRST = 0
          CHGINDX=0
          TEMP= 10.**100
          DO 120 ISUB=1,15
          DO 120 JSUB=1,20
      120 IMV(ISUB,JSUB)=TEMP
          C(P)=2
          DO 130 ISUB=1,40
      130 PSGN(ISUB)=PMEAN(ISUB)=PPIMEAN(ISUB)=0.
          ITEMP = M+R
          ENCODE ( 8, 1106, IELENGTH) ITEMP
1106      FORMAT(I8)
          ENCODE ( 8,1106, IEWIDTH) M
          CALL FMAT      (R,M)
          PRINT 195
141       FORMAT(* COMPUTATIONAL FORM*)
          WRITE ( 6), IFMT1) ((EMATRIX(ISUB,JSUB),JSUB=1,M),ISUB=1,ITEMP)
          PRINT 141
195       FORMAT(/,/,* STANDARD FORM*)
          ITEMP = M - R
          DO 190 I=1, ITEMP
          DO 190 J=1,M
190       IOTEMP(I,J) = EMATRIX(I,IEM(J))
```

```
FTN 1.4             DATE 7/05/66        AT  150515
          WRITE (61,IFMT1) ((IDTEMP(I,J),J=1,M),I=1,ITEMP)
          DO 194 I = 1, ITEMP
          DO 194 J = 1 , M
   194    EMATRIX(I,J) = IDTEMP(I,J)
          PRINT 196
   196    FORMAT(/,/,* PMATRIX*,/)
          DO 197 ISUB=1,IPSIZE
   197    PRINT 198, (PMATRIX(ISUB,JSUB),JSUB=1,IPSIZE)
   198    FORMAT(1X,7I4)
          PRINT 199
   199    FORMAT(/,/,* QMATRIX*,/)
          DO 20198 ISUB=1,IQSIZE
   20198  PRINT 198, (QMATRIX(ISUB,JSUB),JSUB=1,JQSIZE)
          PRINT 250,R
     250 FORMAT(*1 MANUAL CONTROL SIMULATION*,/,
         $0X,*MARK II*,/,/,/,
         $* CONTROL VALUE APPEARS ON METER NUMBER                1*,/,/,
         $* NUMBER OF DECISIONS REMAINING-METER NUMBER           2*,/,/,
         $* CURRENT POSITION APPEARS ON METER NUMBER             3*,/,/,
         $* DISTANCE MEASURE FROM GOAL APPEARS ON METER NUMBER 4*,/, /,
         $* COST INCREMENT APPEARS ON METER NUMBER               5*,/,/,
         $* CUMULATIVE COST APPEARS ON METER NUMBER              6*,/,/,
         $* CURRENT VELOCITY APPEARS ON METER NUMBER*,11X, *7*,/,/,
         $* CURRENT ACCELERATION APPEARS ON METER NUMBER*, 7X,*8*,  /,/,
         $* PARTITIONED DIMENSIONAL MATRIX OF RANK*,12X,I2)
          DO 150, I=1,135
          DO 150, J=1,55
   150    IPLOT (I,J) = 8H
          IPLTCNT = 2
C     200 BLOCK IS THE BRANCH POINT TO START EACH NEW SUBTRAJECTORY.
     200 DO 210 ISUB=1,20
     210 CMV(ISUB)=PMV(ISUB)=PPMV(ISUB)=0.
          ITEMP = (( GOAL(P) - 100) / 10) + 1
          IPLOT(ITEMP,IPLTCNT) = 5H0
          IPLTCNT = IPLTCNT + 1
          L = 1
          NOPI = NS =   LS = KS = 0
          DKS=NKS=TERMC=0
          J=N=K=0
          INTERPL = 0
          CHGINDX = 0
          IF ( P .NE. 1 ) CHGINDX = 2
          DO 220 ISUB=1,8
          DO 220 JSUB=1,20
     220 SMV(ISUB,JSUB)=0.
          CMV(2)= DEC(P)
          CMV(3)=GOAL(P)
          CMV(4)=GOAL(P+1)-GOAL(P)
          CMV(7) = VEL(P)
          CMV(8) = 0.
          DO 230 ISUB=1,40
     230 CSGM(ISUB)=CMEAN(ISUB)=CPIMEAN (ISUB) = 0.
          DO 240 ISUB=1,15
     240 U(ISUB)=0.
```

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FTN 1.4           DATE 7/05/66        AT  150515
          ITEMP = P
    99990 PRINT 251,ITEMP,GOAL(ITEMP),
         $GOAL(ITEMP+1), VEL(ITEMP), ACCEL(ITEMP),
         $                              DEC(ITEMP),A(ITEMP),CMLIM,
         $PSTAR(ITEMP),TCRT,LOWL(ITEMP),UPL(ITEMP),CSI(ITEMP),ETA(ITEMP),
         $V(ITEMP),W(ITEMP)
      251 FORMAT(1H1, 10X,*SUBTRAJECTORY  *,13X,I2,/,/,/,         /,
         $* INITIAL VALUE OF POSITION        *,17X,F15.2,/,/,
         $* DESIRED FINAL VALUE OF POSITION          *,11X,F15.2,/,/,
         $* INITIAL VELOCITY                         *,11X,F15.2,/,/,
         $* INITIAL ACCELERATION                     *,11X,F15.2,/,/,
         $* DECISIONS AVAILABLE TO REACH FINAL VALUE*,21X,I2,/,/,
         $* REFERENCE LEVEL*,33X,F15.2,/,           /,
         $* MEMORY LIMIT*,49X,I2,/,         /,
         $* THRESHOLD COEFFICIENT OF VARIATION*,14X,F15.3,/,          /,
         $* NUMBER OF CONTROL LEVELS TO INITIATE TERMINAL CONTROL*,8X,I2,/,/
         $,* LOWER LIMIT ON FINAL VALUE OF STATE VARIABLE*,4X,F15.2,/,/,
         $* UPPER LIMIT ON FINAL VALUE OF STATE VARIABLE*,4X,F15.2,/, /,
         $* COEFFICIENT OF TRANSFORMATION LAW, CSI*,10X,F15.2,/,      /,
         $* COEFFICIENT OF TRANSFORMATION LAW, ETA*,10X,F15.2,/,/,
         $* COEFFICIENT OF COST INCREMENTS, V*,15X,F15.2,/,       /,
         $* COEFFICIENT OF FINAL MISS DISTANCE, W*,11X,F15.2)
          PRINT 413      ,P
      413  FORMAT (*1SUBTRAJECTORY*,I3,/,/,
         $8X,*CONTROL*,
         $4X,*REMAINING*,
         $6X,*CURRENT*,
         $8X,*DISTANCE*,
         $5X,*INCREMENTAL*,
         $4X,*CUMULATIVE*,
         $7X,*CURRENT*,
         $8X,*CURRENT*,/,
         $8X,*CHOICE*,
         $9X,*DECISIONS*,
         $6X,*POSITION*,
         $7X,*TO GO*,
         $8X,*COST*,
         $11X,*COST*,
         $13X,*VELOCITY*,
         $7X,*ACCELERATION*,/,/,
         $         *          CMV (1)*,8X,
         $*CMV (2)*,8X,
         $*CMV (3)*,8X,
         $*CMV (4)*,8X,
         $*CMV (5)*,8X,
         $*CMV (6)*,8X,
         $*CMV (7)*,8X,
         $*CMV (8)*,8X,
         $/,/)
          ICN = 4
    C   300 BLOCK ESTABLISHES THE CONTROL VALUES.
      300 J=1-M(P)
      310 U(J)=J-10
          J=J+1
```

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FTN 1.4            DATE 7/05/66          AT  150515
          IF(J .LE.10+C(P)) GO TO 310
C       ≠CSUB≠ IS A SUBROUTINE FOR PICKING CONTROL VALUES.
          CALL CSUB(J)
          GO TO 400
380       ICNT = 3
          PRINT 413    ,P
          GO TO 419
C       400 BLOCK IS THE TRANSFORMATION LAWS.
400       CONTINUE
          DO 410 ISUB=1,M
          PPMV(ISUB)=PMV(ISUB)
      410 PMV(ISUB)=CMV(ISUB)
          CMV(1)=U(J)
          CMV(2)=PMV(2)-1.
          CMV(7) = (PMV(7) + ETA(P) * CMV(1)) / ( 1. + CSI(P))
          CMV(3) = PMV(3) + CMV(7)
          CMV(4)=GOAL(P+1)-CMV(3)
          CMV(5)=((CMV(7)-A(P))**2)*V(P)
          CMV(6)=PMV(6)+CMV(5)
          CMV(8) = CMV(7) - PMV(7)
          IF ( IXOUTPUT .NE. 1 )       GO TO 90414
          PRINT 414, CHGINDX, INTERPL,CMLIM, TERMC, H, DS, LS, KS, NOPI, L
         $,ALPHA
414       FORMAT(*0CHGINDX = *,I5,/,
         $*0INTERPL = *,I5,/,
         $*0CMLIM = *,I5,/,
         $*0TERMC = * , I5,/,
         $*0H = *,I5,/,
         $*0DS =*,I5,/,
         $*0LS = *,I5,/,
         $*0KS = *,I5,/,
         $*0NOPI =*,I5,/,
         $*0L =*,I5,/,
         $*0ALPHA = *,     F15.3,1X)
90414     CONTINUE
          ITEMP = (( CMV(3) - 100 ) / 10          ) + 1
          IPLOT(ITEMP,IPLTCNT) = 54B
          IPLTCNT = IPLTCNT + 1
          PRINT 415 ,(CMV(ISUB),ISUB=1,8)
415       FORMAT(1H0,8F15.3)
          ICNT = ICNT + 2
          IF ( ICNT .GE. 54) GO TO      380
419       DO 420 ISUB=1,M
      420 IMV(J,ISUB)=CMV(ISUB)-PMV(ISUB)
          IF ( IFRST .NE. 0 ) GO TO 490
          IF( CHGINDX.GT. 2*C(P)+1) GO TO 1500
440       CALL CSUB(J)
          GO TO 400
490       IFRST = IFRST + 1
          GO TO(440,                 1500) IFRST
C       500 BLOCK - INTERPOLATIONS AT ZERO COST INCREMENTS.
      500 IF(H.NE.1) GO TO 550
          IF(IMV(J,5).GT.0.) N     =N    -2
          N = N + 1
```

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FTN 1.4            DATE 7/05/66         AT 150515
         D = D + 1
         MEAS      =  (N*1.) / (D*1.)
         IF(MEAS    .GE. .5) GO TO 1300
         L=L5+1
         GO TO 4000
     550 IF(CMV(5).NE.0.) GO TO 700
         K=1
     560 INTMV(K)=CMV(K)
         K=K+1
         IF(K.LE.M) GO TO 560
         INTERPL=INTERPL+1
         I=1
         DO 570 ISUB=1,M
     570 SMV(I,ISUB)=INTMV(ISUB)
         K=1
     580 I=CMLIM  + 1
   590   SMV(I,K) = SMV (I-1, K)
         I=I-1
         IF(I.GT.1) GO TO 590
         K=K+1
         IF(K.LE.M) GO TO 580
         IF(INTERPL.GE.CMLIM) GO TO 1000
   C   600 BLOCK - CHOICE OF CONTROL BY MINIMIZING.
     600 ISUB=10-C(P)
         ITEMP= (C(P)*2)+ISUB
         XXMIN= 10.**101
         DO 620 JSUB=ISUB,ITEMP
         IF(IMV(JSUB,5)**2.GE.XXMIN) GO TO 620
         XXMIN=IMV(JSUB,5)**2
         J=JSUB
     620 CONTINUE
         GO TO 400
   C   700 BLOCK - SEARCH ALGORITHM.
     700 IF(IMV(J,5).LT.0.) GO TO 400
         IF(CMV(1).EQ.PMV(1)) GO TO 750
         CALL CSUB(J)
         GO TO 400
   740   CALL CSUB(J)
         GO TO 400
     750 IF(PPMV(5).LT.PMV(5)) GO TO 740
         IF( PPMV(5)+CMV(5).EQ. 2* PMV(5)) GO TO 740
   C   800 BLOCK - INTERPOLATION OF METER READINGS.
     800 ALPHA= (PPMV(5)-CMV(5))/(2.*(PPMV(5)-(2*PMV(5))+CMV(5)))
         K=1
         IF ( ALPHA.GE.0.) GO TO 850
     820 INTMV(K)=PMV(K)+(ALPHA*(PMV(K)-PPMV(K)))
         K=K+1
         IF(K .LE. M) GO TO 820
         GO TO 900
     850 INTMV(K)= PMV(K)+(ALPHA*(CMV(K)-PMV(K)))
         K=K+1
         IF(K .LE.M) GO TO 850
   C   900 BLOCK - STORAGE OF METER READINGS.
   900   CONTINUE
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FTN 1.4          DATE 7/05/66        A1  150515
      IF ( IXOUTPUT .NE. 1 ) GO TO 90900
88840 FORMAT(* INTMV =   *,/,/,(1X,F20.3))
      PRINT 88840, ( INTMV(ISUB),ISUB = 1,20)
90900 CONTINUE
      INTERPL = INTERPL +1
      I=1
      DO 910 KSUB=1,M
  910 SMV(I,KSUB)=INTMV(KSUB)
      K=1
  920 I=CMLIM  + 1
  930 SMV(I,K) = SMV(I-1,K)
      I=I-1
      IF(I .GT.1) GO TO 930
      K=K+1
      IF ( K .LE. M) GO TO 920
      IF (INTERPL .LT. CMLIM) 960, 1000
960   CALL CSUB(J)
      GO TO 400
C   1000 BLOCK - COMPUTE COEFFICIENT OF VARIATION FOR METER READINGS.
 1000 NS = CMLIM
      S=SS=0
      K=1
 1010 I=2
      SS = S = 0
 1020 S=S+SMV(I,K)
      SS=SS+ (SMV(I,K)**2)
      I=I+1
      IF(I .LE. CMLIM+1) GO TO 1020
      VAR(K)=(SS-((S*S)/(NS*1.)))/(NS*1.-1.)
      IF ( IXOUTPUT .NE. 1 ) GO TO 91020
      PRINT 89999, VAR(K),SS,S,NS,K
89999 FORMAT(* VAR = *,F20.4,/,
     $* SS = *,F20.4,/,
     $* S = *,F20.4,/,
     $* NS = *,I5,/,
     $* K = *,I5,/,/)
91020 CONTINUE
      CMEAN(K)= S/(NS*1.)
      CPIMEAN (K) = 1.
      SCFV(K)=VAR(K)/(CMEAN(K)**2)
      IF ( SCFV(K) .LT. 0. ) SCFV(K) = - SCFV(K)
      CFV(K) = SQRTF(SCFV(K))
      IF( CFV(K).GE. PSTAR(P)) GO TO 1100
 1050 NOPI=NOPI+1
      CSGN(K)=1
      WRITE (MT3) K, CMEAN(K)
C   1100 + 1200 BLOCKS - COMPUTE COEFFICIENT OF VARIATION FOR
C       COMBINATIONS OF METER READINGS.
 1100 K=K+1
      IF(K .LE.M) GO TO 1010
      K=M+1
      KSUB = 1
 1110 I=CMLIM  + 1
 1120 TEMP=1.
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FTN 1.4              DATE 7/05/66         AT  150515
            DO 1130 ISUB=1,N
            IF ( SMV(I,ISUB) .EQ. 0.) GO TO 1130
            TEMP=TEMP*(SMV(I,ISUB)**FMATRIX(KSUB,ISUB))
     1130   CONTINUE
            SPICB(I,K)=TEMP
            I=I-1
            IF(I.GT.1) GO TO 1120
            KSUB = KSUB + 1
      1150 K=K+1
            IF(K.LE.2*M-R) GO TO 1110
            IF ( IXOUTPUT .NE. 1 ) GO TO 91160
            PRINT 88920, ((SPICB(ISUB,JSUB),JSUB=1,10),ISUB=1,8)
     88920 FORMAT(*1SPICB = *,/,
           $(10(1X,F12.6)))
     91160 CONTINUE
      1160 NS=CMLIM
            SS=S=0
            K=M+1
      1170 I=2
            S = SS = 0
      1180 S=S+SPICB(I,K)
            SS=SS+ (SPICB(I,K)**2)
            I=I+1
            IF(I .LE. CMLIM+1) GO TO 1180
      1200 VAR(K)=(SS-((S*S)/(NS*1.)))/((NS*1.)-1.)
            CPIMEAN(K) = S/(NS*1.)
            SCFV(K)=VAR(K)/(CPIMEAN(K)**2)
            IF ( SCFV(K) .LT. 0. ) SCFV(K) = - SCFV(K)
            CFV(K)=SQRTF(SCFV(K))
            IF ( IXOUTPUT .NE. 1 ) GOTO 91200
            PRINT 89999, VAR(K),SS,S,NS,K
     91200 CONTINUE
            IF(CFV(K).GE.PSTAR(P)) GO TO 1220
            NOPI=NOPI+1
            CSGN(K)=1
            ITEMP = K - M
            DO 1218 JSUB =1,M
     1218   IDTEMP(ITEMP,JSUB) = EMATRIX(ITEMP,     JSUB )
            WRITE (MT4) CPIMEAN(K), (IDTEMP(ITEMP,JSUB),JSUB=1,M)
      1220 K=K+1
            IF(K .LE. 2*M-R) GO TO 1170
            IF ( NOPI.LE. 0) GO TO 1250
            L=1
            GO TO 4000
     1250   CALL CSUB(J)
            GO TO 400
     C    1300 + 1400 BLOCKS - CHOICE OF CONTROL ON HEURISTIC.
     1300   CONTINUE
            IF ( IXOUTPUT .NE. 1 ) GO TO 91300
                                 PRINT 88810,CPI ,CMV(KS),CMEAN(KS),KS
     88810 FORMAT(     /,/,* CPI = *, F 20.3,
           $*  CMV(KS) = *,F10.3,*  CMEAN(KS) = *,F10.3,*  KS = *,I2)
            PRINT 88820,(IPI(JSUB),JSUB=1,15)
     88820 FORMAT(/,/,/, * IPI = *,/,/,  (1X,F20.3))
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FTN 1.4            DATE 7/05/66        AT  15.51S
            PRINT 88860, CPIMEAN(KS),EPI(KS),KS
      88860 FORMAT(/,/,* CPIMEAN(KS) = *,E20.3,*  EPI(KS) = *,E20.3,
           S*    KS = *,I2)
      91300 CONTINUE
            IF(CPIMEAN(KS)-EPI(P).LE. CPI      .AND.
           $CPIMEAN(KS)+EPI( P).GE. CPI      ) GO TO 1400
            J=10-C(P)
       1320 IF(IPI(J).NE.0.) GO TO 1330
      1325  PERLY(J)= -10.**100
            GO TO 1350
       1330 ESTOPI(J)=(CPIMEAN(KS)-CPI      )/IPI(J)
            IF(ESTOPI(J) .LE. 0.) GO TO 1325
            PERLY(J)=CMV(2)-ESTOPI(J)
            IF (PERLY(J).LE.0.) PERLY(J)=-10.**100
       1350 J=J+1
            IF(J .LE. 10+C(P)) GO TO 1320
            ISUB =10-C(P)
            ITEMP=(C(P)*2)+ISUB
            XXMIN=-10.**101
            DO 1370 JSUB=ISUB,ITEMP
            IF(PERLY(JSUB) .LT. XXMIN) GO TO 1370
            XXMIN=PERLY(JSUB)
            J=JSUB
       1370 CONTINUE
            GO TO 400
       1400 ISUB= 10-C(P)
            ITEMP=(C(P)*2)+ISUB
            XXMIN = 10.**101
            DO 1420 JSUB=ISUB,ITEMP
            IF ( IPI(JSUB) **2 .GE. XXMIN ) GO TO 1420
            XXMIN=IPI(JSUB) ** 2
            J=JSUB
       1420 CONTINUE
            GO TO 400
       1500 IF( CMV(2).GT.0.) GO TO 1510
            IF(TERMC.GT.0) GO TO 1800
            PRINT 1505, P
      1505  FORMAT(* SUBTRAJECTORY *,I2,* ENDED USING SEARCH PROCEDURE.*,/,/)
            GO TO 1800
       1510 IF(GOAL(P+1)-LOWL(P).LE.CMV(3).AND.
           $GOAL(P+1)+UPL(P).GE.CMV(3)) GO TO 1890
      C     7000 BLOCK - DETERMINS IF TERMINAL CONTROL SHOULD BEGIN.
      7000  T = 0
            J SUB = 10 - C(P)
            CV = JSUB - 11
      7010  RD = 1
            CV = CV + 1
            EFFY = ( ETA(P) * CV) / CSI (P)
      7020  FACTOR = ( 1. - ( 1. / (( 1. + CSI(P)) ** RD)))
            HCMV = ( CMV(3) + ((RD) * (EFFY ))) + (( 1. / CSI(P))* (FACTOR)
           $ * ( CMV(7) - EFFY ))
            RD = RD + 1
            IF ( RD * 1. .LE. CMV(2) + 1. ) GO TO 7020
            IF ( GOAL(P+1) .GT. CMV(3) .AND.
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FTN 1.4          DATE 7/05/66        AT  150515
     $      HCMV .GT. GOAL (P+1)) GO TO 7025
      IF ( GOAL(P+1) .LE. CMV(3) .AND.
     $      HCMV .LE. GOAL(P+1)) GOT O 7025
      ERLY(JSUB) = -10.**100
      ESTOG(JSUB) = HCMV - GOAL(P+1)
      GO TO 7027
 7025 T = T + 1
      ERLY(JSUB) = HCMV - GOAL(P+1)
 7027 IF ( IXOUTPUT .NE. 1 ) GO TO 7040
      PRINT 7030, EFFY, FACTOR, HCMV, ERLY(JSUB), T
 7030 FORMAT(* EFFY = *,F20.5,/,
     $* FACTOR = *,F20.5,/,
     $* HCMV = *,F20.5,/,
     $* ERLY(JSUB) = *,F20.5,/,
     $* T = *,I3,/)
 7040 JSUB = JSUB + 1
      IF ( JSUB .LE. 10 + C(P)) GO TO 7010
      IF ( T .LE. TCRT ) 7050, 7060
 7050 TERMC = TERMC + 1
      IF ( T .LE. 0 ) GO TO 7090
      GO TO 7070
 7060 TERMC = 0
      IF ( CMV(2) .LE. FINTERM ) GO TO 7050
      CPI = CMV(KS) / CMEAN(KS)
      GO TO 500
 7070 ISUB = 10 - C(P)
      ITEMP = ( C(P) * 2 ) + ISUB
      XXMIN = 10. ** 101
      DO 7080 KSUB = ISUB , ITEMP
      IF ( ERLY(KSUB) **2.GE. XXMIN ) GO TO 7080
      XXMIN = ERLY(KSUB) ** 2
      J = KSUB
 7080 CONTINUE
      GO TO 400
 7090 ISUB = 10 - C(P)
      ITEMP = ( C(P) * 2 ) + ISUB
      XXMIN = 10. ** 101
      DO 7095 KSUB = ISUB, ITEMP
      IF ( ESTOG(KSUB) ** 2 .GE. XXMIN ) GO TO 7095
      XXMIN = ESTOG(KSUB) ** 2
      J = KSUB
 7095 CONTINUE
      GO TO 400
C   1800 BLOCK - DETERMINE PENALTY AND TOTAL COST.
 1800 ALOWL = GOAL(P+1) - LOWL(P)
      AUPL = GOAL(P+1)     +UPL(P)
      IF(ALOWL .LE.CMV(3).AND.  .
     $AUPL.GE. CMV(3)) GO TO 1810
      IF(CMV(3).LT.ALOWL) GO TO 1805
      TCST=        W(P)*((CMV(3)-AUPL)**2)
      PRINT 1804, TCST
 1804 FORMAT(/,/,*0PENALTY IS*,F20.3)
      TCST = TCST + CMV(6)
      PRINT 1806, TCST
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FTN 1.4            DATE 7/05/66        AT  150515
 1806  FORMAT(* TOTAL COST IS *,F20.2)
       GO TO 1820
  1805 TCST=         W(P)*((ALOWL-CMV(3))**2)
       PRINT 1804, TCST
       TCST = TCST + CMV(6)
       PRINT 1806, TCST
       GO TO 1820
 1810  TCRT = TCST - 1
       IF ( TCRT .LT. 1) TCRT = 1
       TCST = CMV(6)
       PRINT 1807 , TCST
 1807  FORMAT(/,/,*0PENALTY IS 0*,/,* TOTAL COST IS *,F20.2)
  1820 K=0
       ISW=1
       GO TO 2000
 1690  IF (TERMC.GT.0) GO TO 1900
       IF ( IMV(J,5) .LT. 0.) GO TO 400
       CALL CSUB(J)
       GO TO 400
C    1900 BLOCK - CHOICE OF CONTROL FOR TERMINAL OPERATION.
  1900 ISUB=10-C(P)
       ITEMP=(C(P)*2)+ISUB
       XXMIN=10.**101
       DO 1920 JSUB=ISUB,ITEMP
       IF(IMV(JSUB,4)**2.GE.XXMIN) GO TO 1920
       XXMIN=IMV(JSUB,4)**2
       J=JSUB
  1920 CONTINUE
       GO TO 400
C    2000 BLOCK - DETERMINATION OF HEURISTICS AND CONFIDENCE MEASURES
C      AND MEMORY LIMIT FOR NEXT SUBTRAJECTORY.
 2000  CONTINUE
       IF ( I2000 .EQ. 1 ) GO TO 2003
       I2000 = 1
       END FILE MT3
       END FILE MT4
       REWIND MT 3
       REWIND MT 4
       IF ( INTERPL .LT. CMLIM ) GO TO 2009
       ICNT = 3
       PRINT 2001,P
 2001  FORMAT(*1METERS READING CONSTANT FOR SUBTRAJECTORY*,I3,/,/)
 2002  READ (MT3) IDUM1, IDUM2
       IF ( EOF, MT3) 2005,  2004
 2004  PRINT 1061, IDUM1, IDUM2
       ICNT = ICNT + 1
       IF ( ICNT .GE. 54 ) GO TO 2017
 1061  FORMAT(*0WHEN COST INCREMENTS ARE MINIMAL, METER *,
      $ I2,* READS *,F17.2)
       GO TO 2002
 2005  READ (MT4) IDUM1, (IDUM(ISUB),ISUB=1, M)
       IF ( EOF,MT4) 2008,  2006
 2006  PRINT 1205, IDUM1, ( IDUM(ISUB),ISUB=1, M)
       ICNT = ICNT + 1
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FTN 1.4           DATE 7/05/66        AT  150515
      IF ( ICNT .GE. 54 ) GO TO 2019
 1205 FORMAT(*0WHEN COST INCREMENTS ARE MINIMAL, THE FOLLOWING COMBINAT*
     $,*ION OF METER READINGS EQUALS *,F17.2
     $,/,26I5)
      GO TO 2005
 2008 REWIND MT3
      REWIND MT4
 2009 JSW = 0
      K=K+1
      IF(K.GT.2*M-R) GO TO 2050
      IF(PSGN(K)*CSGN(K).NE.1.) GO TO 2000
      TEMP = ( 1. + DS ) / ( 2. + DS )
      IF(K.EQ.KS) GO TO 2018
 2010 IF(ISW.LE.2) ISW=2
      IF ( L .LE. M .AND. PMEAN(K).EQ. CMEAN(K)) GO TO 2020
      IF ( K .GT. M .AND. PPIMEAN(K) .EQ. CPIMEAN(K)) GO TO 2020
      PRINT 2015,K
 2015 FORMAT(*0DIMENSIONLESS PARAMETER NUMBER *,I2,
     $* IS INVARIANT WITHIN BUT NOT BETWEEN SUBTRAJECTORIES *)
      GO TO 2000
 2017 PRINT 2001
      ICNT = 3
      GO TO 2002
 2019 PRINT 2001
      ICNT = 3
      GO TO 2005
 2018 PRINT 2005, K, TEMP
 2005 FORMAT(*0CONFIDENCE MEASURE OF HEURISTIC BASED ON *,
     $*DIMENSIONLESS PARAMETER NUMBER *,I2,* IS EQUAL *,
     $*TO *,F20.4)
      JSW = 1
      GO TO 2010
 2020 IF ( JSW .EQ. 1 ) ISW = 3
      PRINT 2025,K
 2025 FORMAT(*0DIMENSIONLESS NUMBER *,I2,* IS INVARIANT *,
     $*WITHIN AND BETWEEN SUBTRAJECTORIES *)
      GO TO 2000
 2050 JSUB=2*M-R
      DO 2060 ISUB=1,40
      PPIMEAN(ISUB)=CPIMEAN(ISUB)
      PMEAN (ISUB) = CMEAN(ISUB)
 2060 PSGN(ISUB) = CSGN(ISUB)
      IF (ISW .EQ.3) CMLIM=CMLIM-2
      IF(ISW.EQ.2) CMLIM=CMLIM-1
      IF(CMLIM.LT.1)CMLIM=1
      DO 2070 ISUB = 1, 135
      IF ( ISUB .EQ. ((GOAL(P+1) - 100) / 10) + 1 ) GO TO 2070
      IF ( IPLOT(ISUB,DEC(P) + 2).EQ. 8H         )
     $IPLOT(ISUB,DEC(P) +2 ) = 0
 2070 CONTINUE
      ITEMP = (( GOAL(P+1) - 100) / 10) + 1
      IPLOT(ITEMP,DEC(P)+2) = 67B
      IF ( CMV(3) .EQ. 0.) IPLOT(ITEMP,DEC(P)+2 ) = 54B
      ITEMP = 110
```

```
FTN 1.4             DATE 7/05/66        AT 150515
      DO 2075 ISUB=2,135
      IPLOT(ISUB,1) = ITEMP
2075  ITEMP = ITEMP + 10
      ITEMP = DEC(P)
      DO 2080 ISUB = 2, 55
      IPLOT(1,ISUB) = ITEMP
      ITEMP = ITEMP - 1
      IF ( ITEMP .LT. 0 ) GO TO 2085
2080  CONTINUE
2085  ITEMP = DEC(P)     +2
      JTEMP =  136
      PRINT 2087,P
2087  FORMAT(*1POSITION AS A FUNCTION OF REMAINING DECISIONS*,
     $* FOR SUBTRAJECTORY*,I3,/,/)
      DO 2090 ISUB = 2 , 135
      JTEMP = JTEMP - 1
2090  PRINT 2095, (IPLOT(JTEMP,JSUB),JSUB=1,ITEMP)
2095  FORMAT(1X,I4, 55(2X,R1))
      PRINT 2086, ( IPLOT(1,ISUB),ISUB=2,ITEMP)
2086  FORMAT(1X,4X, 55(1X,I2))
      IPLTCNT = 2
      DO 2096 ISUB = 1,135
      DO 2096 JSUB =1,55
2096  IPLOT (ISUB,JSUB) = 8H
      P=P+1
      I2000 = 0
      IFRST = 1
      REWIND MT3
      REWIND MT4
      IF(P.LE.PMAX) GO TO 200
      PRINT 3000
 3000 FORMAT(*1END OF TRAJ.*)
      CALL EXIT
4000  K = 1
4010  IF ( PRIR(K) .EQ. L ) GO TO 4015
      K = K + 1
      GO TO 4010
4015  IF ( CSGN(K) .EQ. 1 ) GO TO 4017
4016  L = L + 1
      IF ( L .LE. 2*M-P) GO TO 4000
      H = 0
      CALL C SUB (J)
      GO TO 400
4017  H = 1
      DS = CMV (2)
      LS = L
      KS = K
      N = 1
      D = 2
      MEAS = 0.5
      IF ( KS .GE. M + 1 ) GOTO 4045
      IF ( CMEAN (KS) .EQ. 0. ) GO TO 4016
      CPI = CMV(KS) / CMEAN(KS)
      J = 10 - ((P)
```

```
FTN 1.4              DATE 7/05/66        AT  150515
     4030  IPI(J) = IMV(J,KS ) / CMEAN(KS)
           J = J + 1
           IF ( J .GT. C(P) + 10) 1300,  4030
     4045  TEMP = 1.
           KSUB = KS - M
           DO 4050 ISUB = 1, M
           IF ( CMV(ISUB) .EQ. 0.) GO TO 4050
           TEMP = TEMP * (CMV(ISUB)** EMATRIX(KSUB,ISUB) )
     4050  CONTINUE
           CPI = TEMP
           J = 16 - C(P)
           TEMP = 0.
     4055  DO 4060 ISUB = 1,M
     4060  TEMP = TEMP +((EMATRIX(KSUB,ISUB)*IMV(J,ISUB)) / CMV(ISUB))
           IPI(J)     = CPI * TEMP
           J = J + 1
           IF ( J .GT. 10 + C(P)) GO TO 1300
           GO TO 4055
           END
```

```
FTN 1.4            DATE 7/05/66        AT  150515
          SUBROUTINE CSUB(J)
          COMMON/CCOM/CHGINDX,CTABLE(8)
          DATA ( CTABLE=0,0,1,-1,2,-2,1,-1),(CHGINDX=0)                     $
          TYPE INTEGER CHGINDX,CTABLE
          J=CHGINDX-(CHGINDX/8)*8+1                                          $
          J=CTABLE(J)    +10
          CHGINDX=CHGINDX+1
          RETURN
          END
```

```
FTN 1.4            DATE 7/05/66          AT  150515
      SUBROUTINE EMAT(R,M)
C THIS PROGRAM HAS BEEN CHECKED OUT AND IS ≠OK≠  602066
C     SUBROUTINE EMAT COMPUTES EMATRIX= (-QPINV,K*I) WHERE Q+P ARE
C     FURNISHED THRU LABELED COMMON, DIMENSIONS ARE P(R,R), Q(M-R,R)
C     AND I (M-R,M-R). PINV IS COMPUTED FROM P AND HENCE P MUST BE
C     NON-SINGULAR. K IS A POSITIVE INTEGER CONSTANT WHICH IS THE
C     SMALLEST INTEGER WHICH WILL ALLOW PINV TO HAVE ALL INTEGER
C     ENTRIES
      COMMON/ECALC/P(7,7),Q(19,7),E(19,26)
      DIMENSION ITEMP(7,14),IPRIME(5)
      DATA (IPRIME=2,3,5,7,11)
      INTEGER P,Q,E,R
      DO 1 J=1,R
      DO 1 I=1,R
      ITEMP(I ,J)=P(I,J)
    1 ITEMP(I,J+R)=0
      DO 2 I=1,R
    2 ITEMP(I,I+R)=1
      IR=R+R
C     CONSTRUCT ITEMP = (P,I)
      DO 3 IP=1,R
      IPIV=ITEMP(IP,IP)
C     REDUCE P TO DIAGONAL MATRIX BY INTEGER ROW TRANSFORMATIONS.
      IF(IPIV) GO TO 20
      DO 22 I=IP,R
      IF(ITEMP(I,IP)) GO TO 21
   22 CONTINUE
      GO TO 19
   21 DO 23 J=IP,IR
   23 ITEMP(IP,J)=ITEMP(IP,J)+ITEMP(I,J)
      IPIV=ITEMP(IP,IP)
   20 DO 3 I=1,R
      IF(I.EQ.IP)GO TO 3
      IPPIV=ITEMP(I,IP)
      DO 5 J=1,IR
    5 ITEMP(I,J)=ITEMP(I,J)*IPIV-IPPIV*ITEMP(IP,J)
    3 CONTINUE
C     COMPUTE LEAST COMMON POSITIVE MULTIPLE OF DIAGONAL ELEMENTS
      IPROD=1
      DO 4 I=1,R
      IPIV=ITEMP(I,I)
      IF((IPROD/IPIV)*IPIV.EQ.IPROD) GO TO 4
      IPROD=IPROD*IPIV
    4 CONTINUE
      IF(IPROD.LT.0) IPROD=-IPROD
      IRP1=R+1
C     MULTIPLY PINV BY ROW BY LCM.
      DO 6 I=1,R
      MULT=IPROD/ITEMP(I,I)
      ITEMP(I,I)=IPROD
      DO 6 J=IRP1,IR
    6 ITEMP(I,J)=ITEMP(I,J)*MULT
      DETERMINE IF P IS SINGULAR.
      IF(IPIV.EQ.0)GO TO 19
```

```
FTN 1.4          DATE 7/05/66        AT  15^515
      IP=1
C     REMOVE FACTORS OF PINV AND K.
    9 IPIV=IPRIME(IP)
   13 IF(((ITEMP(1,1)/IPIV)*IPIV.EQ.ITEMP(1,1)) GO TO 7
   14 IP=IP+1
      IF(IP.GT.5) 8,9
    7 DO 10 J=IRP1,IP
      DO 10 I=1,R
      IF(((ITEMP(I,J)/IPIV)*IPIV.EQ.ITEMP(I,J)) 10,11
   10 CONTINUE
      ITEMP(1,1)=ITEMP(1,1)/IPIV
      DO 12 J=IRP1,IR
      DO 12 I=1,R
   12 ITEMP(I,J)= ITEMP(I,J)/IPIV
      GO TO 13
   11 IF(IABS(ITEMP(I,J)).LT.IPIV)8,14
    8 IRP =M-R
C     FORM -Q*PINV
      DO 15 I=1,IRP
      DO 15 J=1,R
      IPIV=0
      DO 16 K=1,R
   16 IPIV=IPIV-Q(I,K)*ITEMP(K,J+R)
   15 E(I,J)=IPIV
C     ADD K*I TO EMATRIX
      DO 17 I=1,IRP
      DO 17 J=IRP1,M
   17 E(I,J)=0
      DO 18 IP=1,IRP
   18 E(IP, R   +IP)=ITEMP(1,1)
C     RETURN WITH EMATRIX = (-Q*PINV, K*I)
      RETURN
C     EXIT IF P IS SINGULAR.
   19 PRINT 102,R,((ITEMP(I,J),I=1,R),J=1,IR)
  102 FORMAT(27H ***P-MATRIX IS SINGULAR,R=,I2/(7I10))
      CALL EXIT
      END
```

APPENDIX B

ANALYSIS OF ALTERNATIVE MARK I STRATEGIES

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ANALYSIS OF ALTERNATIVE MARK I STRATEGIES

The results presented in this report show excellent agreement between the subject median fuel cost and the Mark I fuel cost. Because of the simplicity of the Mark I test problems, there arises the possibility that the good agreement was "forced" by the structure of the problems. To gain some insight to this question several hypothetical alternative strategies were used to solve the 23 control problems. The 23 fuel costs obtained with each of these strategies are compared with the subject median costs by means of the correlation coefficient.

Selection of Controls Using a Random Strategy

To obtain an underestimate of the correlation coefficients associated with alternative strategies it is supposed that a hypothetical subject chooses his controls at random. Specifically, it is assumed that at each decision time, one of the controls, -2, -1, 0, 1, 2, is selected, with each of the five controls having a probability of 1/5 of being selected.

The sequence of random choices was obtained by entering Hald's table of random sampling numbers.[16] As a result of a random process (coin toss), the table was entered on page 93, row 26, column 10, and the two digit numbers shown in the table were read downward, beginning with 24, 09, 28, etc. The random number intervals (0,19), (20,39), (40,59), (60,79), and (80,99) were associated with the controls -2, -1, 0, 1, and 2, respectively. The 383 random numbers gave ordered observed frequencies of (76,63,84,87,73) for the ordered sequence of control values (-2, -1, 0, 1, 2). Based on an expected frequency of 383/5 = 76.6, this yields a chi-square value of 4.72 with four degrees of freedom. Because this is less than the 95 percent fractile, 9.49, the sequence was accepted as a random sequence. No other tests for randomness of the sequence were made.

Analysis With Miss-Distance Penalty Excluded

Table B-1 shows the subject median cost and the random strategy cost for each sub-trajectory. Column 3 shows the total fuel costs with the miss-distance penalty included. This penalty was equal to 100 times the square of the miss-distance at the end of each sub-trajectory and is the same penalty as that used for the 14 subjects. The correlation coefficient between Columns 2 and 3 is found to be 0.268.

The statistical significance of a correlation coefficient, r, based on n pairs of observations, may be tested using the statistic,

$$t = \frac{r}{\sqrt{1 - r^2}} \sqrt{f} ,$$

which has a t-distribution with $f = n-2$ degrees of freedom when the true correlation ρ is equal to zero. With $r = 0.268$, and $f = 21$, the computed value of t is found to be 1.26. For a one-sided test (for positive correlation) the tabulated value of t-variate is found from Hald's tables (16) to be 1.721. Thus, $r = 0.268$ is statistically equal to zero, and the hypothesis of zero correlation between subject behavior and the random strategy is not rejected.

TABLE B-1. SUBJECT MEDIAN COST AND RANDOM STRATEGY COST FOR EACH SUBTRAJECTORY

(Kilo Units of Fuel)

		Random Strategy	
Subtrajectory Number	Subject Median	With Miss-Distance Penalty	Without Miss-Distance Penalty
1	35.150	397.500	397.500
2	30.400	436.400	76.400
3	1.635	369.470	9.470
4	76.000	5712.000	2102.000
5	23.850	2017.500	57.500
6	9.101	163.956	3.956
7	8.046	648.446	8.446
8	95.250	2201.200	511.200
9	5.180	1478.140	38.140
10	21.575	9814.500	204.500
11	128.950	3625.300	15.300
12	27.225	280.900	120.900
13	65.200	696.900	386.900
14	80.500	109.000	19.000
15	2.175	251.500	1.500
16	17.600	43.600	43.600
17	11.700	58.840	18.840
18	22.600	651.000	11.000
19	1.330	1443.720	3.720
20	44.400	142.400	52.400
21	20.650	2299.650	49.650
22	4.024	44.577	4.577
23	41.700	1710.700	20.700

The low value of the correlation coefficient supports the assertion that the controls were not randomly selected, and also suggests that the high correlation between the subject median fuel costs and Mark I fuel costs was not "forced".

Because the miss-distance penalty was quite large relative to the penalties for deviations from the reference velocity, it is of some interest to compute the correlation between the subject median costs and the random strategy costs when the miss-distance penalties are omitted. The resulting costs are shown in Column 4 of Table B-1. The correlation coefficient between these costs and the subject median costs is found to have a value of 0.384. Here the computed t-value is equal to 1.91. This exceeds the tabulated value of 1.721 so the correlation is judged to be statistically significant. However, the value of the correlation coefficient is too small to be of practical significance in relating subject median cost to that of a random strategy without miss-distance penalties.

Three Strategies Independent of Incremental Costs

Fast Approach. In this section we consider three strategies which are nonrandom, but do not take any account of the incremental fuel costs. The first strategy, the fast approach, consists of approaching the desired final velocity as rapidly as possible and maintaining the final velocity for the remaining time. This strategy was observed for most subjects, particularly in the early problems of the sequence. In effect, the subject first determines whether the problem is "feasible". He may determine this by actually reaching the final velocity as early as possible and spending the remaining time in attempting to minimize fuel.

Table B-2, Column 3, shows the total fuel costs obtained with the fast approach strategy. The correlation coefficient between these costs and the subject median costs is found to be equal to 0.863.

The t-test used above shows that this correlation coefficient is statistically greater than zero. More interesting is the question of whether this correlation coefficient is statistically smaller than that obtained between the subject median fuel cost and the Mark I fuel cost, given by r = 0.916. Using Fisher's z-transform, the quantity

$$u = (z-\xi)\ \sqrt{u-3} \quad ,$$

where

$$z = (1/2)\ \ln[(1+r)/(1-r)]$$

and

$$\xi = (1/2)\ \ln[(1+\rho)/(1-\rho)] + \rho/(2)(n-1) \quad ,$$

is approximately normally distributed with zero mean and unit variance. With r = 0.916, these expressions may be solved by trial to obtain an approximately one-sided lower confidence interval for ρ. This calculation yields (0.81, 1.00) as a 95 percent confidence interval for ρ. Because 0.863 lies in this interval, it is not significantly smaller than 0.916 at the 5 percent level of significance. Thus, based on the observed values of correlation coefficients, the Mark I model and the fast approach strategy give statistically equivalent descriptions of the subject median costs.

TABLE B-2. TOTAL FUEL COSTS FOR SUBJECT MEDIAN AND THREE STRATEGIES INDEPENDENT OF INCREMENTAL COSTS

(Kilo Units of Fuel)

Subtrajectory Number	Subject Median	Fast-Approach Strategy	Slow-Approach Strategy	Straight-Line Strategy
1	35.150	25.200	332.400	39.000
2	30.400	34.600	236.200	58.600
3	1.635	2.850	16.300	4.020
4	76.000	138.000	398.000	108.500
5	23.850	93.800	15.300	36.600
6	9.101	36.256	4.896	15.616
7	8.046	25.966	8.446	11.686
8	95.250	77.800	385.800	180.500
9	5.180	6.010	24.730	12.650
10	21.575	47.000	47.000	26.950
11	128.950	242.400	110.400	145.700
12	27.225	53.000	109.000	140.700
13	65.200	121.500	177.500	134.800
14	80.500	242.000	50.000	155.000
15	2.175	6.240	1.920	4.980
16	17.600	21.600	56.800	28.600
17	11.700	21.120	21.120	21.120
18	22.600	85.200	18.000	51.600
19	1.330	4.180	0.820	2.660
20	44.400	90.000	38.800	75.600
21	20.650	67.650	16.450	43.250
22	4.024	12.415	3.979	9.271
23	41.700	112.800	36.000	80.100

<u>Slow Approach</u>. As a contrasting strategy, a slow approach strategy was evaluated with costs given in Column 4 of Table B-2. In this case the strategy consists of keeping the initial velocity unchanged as long as possible before approaching the desired final value as rapidly as possible. The calculated value of the correlation coefficient for this strategy is found to be equal to 0.580. In this case the correlation coefficient does not lie in the interval, (0.81, 1.00), so that the correlation is statistically smaller than that obtained with the Mark I model.

<u>Straight-Line Approach</u>. A strategy intermediate to the fast approach and slow approach consists of approaching the final velocity in a linear manner. To evaluate the correlation coefficient associated with a straight line approach, the controls were chosen to keep the value of the current velocity strictly higher than the "straight-line" velocity, if the velocity were to be increased between the initial and final points, and strictly lower than the "straight-line" velocity, if the velocity were to be decreased. An approximation to the strategy was observed for several subjects. One subject making hand computations in a pilot study, requested a ruler, drew the appropriate line, and then attempted to keep his current velocity on the line.

Table B-2 shows the costs resulting from this strategy in Column 5. The correlation coefficient between this column and the subject median cost in Column 2 is found to be equal to 0.883. By the same argument as given above, although this correlation coefficient is smaller than that obtained with the Mark I model, it is not smaller by a statistically significant amount.

In summary, these results show that even for strategies which do not attempt to minimize the incremental costs, this experiment was not capable of showing statistically significant differences between the relevant correlation coefficients. This insensitivity may result from the fact that only 14 subjects were tested. With $(\xi - z)\sqrt{n-3} = -1.64$ and ξ and z corresponding to correlation coefficients of 0.883 and 0.916, it is found that the required number of subjects would be nearly 100 before statistical significance at the five percent level could be demonstrated between these correlation coefficients. This insensitivity is not considered a detraction because the primary objective of this research consists of predicting verbal heuristics, not total fuel consumption.

Three Strategies Dependent Upon Incremental Costs

Most subjects appeared to base their control choices at least partly on incremental costs. In this section we consider three cost-dependent strategies. All three strategies yield correlation coefficients in excess of 0.90 with the subject median cost.

Absolute Minimum Cost. The first cost-dependent strategy consists of making those control choices which yield the absolute minimum fuel cost. This is the mathematical optimum and yields the lowest possible cost. In the first 13 problems the reference level, at which fuel cost was minimal, could always be reached by changing the initial velocity in the direction of the desired final velocity. Several subjects learned to expect this characteristic of the control problems and obtained scores equal, or nearly equal, to the mathematical minimum in the early problems.

Table B-3, Column 3, shows the fuel costs for the absolute minimum strategy. The correlation coefficient between these costs and the subject median costs, shown in Column 2, is found to be 0.909.

Expected Minimum Cost. For problems 14 through 23, the direction of velocity change required to minimize fuel is independent of the final velocity. This independence requires the subject to "search" for the proper change in velocity to yield decreasing costs. The search is unavoidable and gives rise to a second cost-dependent strategy. In this strategy an expected minimum cost was computed as follows. Suppose that controls y and -y are selected at the beginning of a problem. The velocity resulting from the first choice is given by $v_1 = v_0 + 10y$. The second choice yields $v_2 = v_1 - 10y = v_0$. Thus, the current velocity is again equal to the initial value after the second choice. The costs associated with these choices depend on the location of the reference level V. If the first choice of y yields a velocity closer to the reference level than the initial velocity, then the cost increment is given by

$$C(y) = A\left\{(V-v_1)^2 + (V-v_2)^2\right\},$$

$$C(y) = A\left\{(V-V_0-10y)^2 + (V-v_0)^2\right\},$$

or

$$C(y) = A\left\{2(V-v_0)^2 - 20y(V-v_0) + 100y^2\right\}.$$

The sign of the middle term in the bracket is positive or negative, depending on the sign of $(V-v_0)$. Under the assumption that these signs are equally likely for given values of V and v_0, it follows that this conditional expected cost is given by

TABLE B-3. TOTAL FUEL COSTS FOR SUBJECT MEDIAN, ABSOLUTE MINIMUM COSTS, EXPECTED MINIMUM COSTS, AND COMPOSITE COSTS

(Kilo Units of Fuel)

Subtrajectory Number	Subject Median	Absolute Minimum	Expected Minimum	Composite Absolute-Expected Minimum
1	35.150	12.400	32.500	12.400
2	30.400	9.400	25.700	9.400
3	1.635	0.660	1.950	0.660
4	76.000	20.000	56.500	20.000
5	23.850	8.400	9.300	8.400
6	9.101	3.916	4.110	3.916
7	8.046	3.086	3.715	3.086
8	95.250	69.000	126.900	69.000
9	5.180	3.210	6.600	3.210
10	21.575	17.000	27.050	17.000
11	128.950	97.900	103.000	97.900
12	27.225	17.000	31.450	17.000
13	65.200	33.000	57.300	33.000
14	80.500	28.000	32.500	32.500
15	2.175	0.600	0.870	0.870
16	17.600	4.000	14.200	14.200
17	11.700	2.880	6.840	6.840
18	22.600	16.800	17.400	17.400
19	1.330	0.700	0.760	0.760
20	44.400	14.400	22.000	22.000
21	20.650	14.050	15.150	15.150
22	4.024	2.335	2.976	2.976
23	41.700	36.000	36.300	36.300

$$E[C(y)] = A \left\{2(V-v_0)^2 + 100y^2\right\} .$$

Although the choice y=0 will minimize the expected cost, this choice will not yield the direction of the reference level. Thus, the minimization of the expected cost is taken over the set y = -2, -1, 1, 2 and yields y = 1 or y = -1 as equivalent optimal first control choices with a conditional expected cost of

$$C = A \left\{2(V-v_0)^2 + 100\right\} .$$

That is, the conditional expected cost is minimized and the direction of the reference level is determined by the choice of either y = 1, y = -1, or y = -1, y = 1, for the first two control choices.

This procedure for the first two control choices has the effect of reducing the number of decisions available for each control problem by 2. Moreover, each problem is thereby reduced to the minimum cost case considered above, provided that at least two decision intervals are available at the reference level in the minimum cost case. This provision holds for 19 of the 23 control problems. For the four exceptional problems, the above increase in cost over the minimum cost strategy is a good approximation to the expected minimum cost. Thus, the above cost was calculated and added to the minimum cost to obtain the costs associated with the expected minimum strategy given in Column 4 of Table B-3.

The correlation coefficient between the subject median costs and the costs obtained from the expected minimum strategy is found to be 0.907. This is nearly identical to that obtained from the absolute minimum strategy.

Composite Absolute and Expected Minimum Cost. Because a search was required to determine the direction of the reference level from problems 14 through 23, it would be expected that a better strategy to associate with the subject median costs would consist of a composite strategy consisting of the absolute minimum strategy for problems 1 through 13, and the expected minimum strategy for problems 14 through 23. These costs are shown in Column 5 of Table B-3. The correlation coefficient between the subject median costs and the costs obtained from the composite strategy is found to be equal to 0.918. This is the highest correlation coefficient found between the subject median cost and any of the strategies considered. It exceeds the Mark I correlation with subject median costs by 0.002.

Summary of Correlation Results for Subject Median Costs

Table B-4, Column 2, shows a listing of the correlation coefficients between the subject median costs, the Mark I costs, and the eight strategies described above. In summary, the table shows correlation coefficients ranging from the random strategy with miss-distance penalty ($r = 0.268$) to the composite strategy ($r = 0.918$). The Mark I model yields the second highest correlation ($r = 0.916$), and this is followed by the absolute minimum strategy ($r = 0.909$) and the expected minimum strategy ($r = 0.907$). These incremental-cost strategies all yield higher correlation coefficients than those which are independent of the cost increments: the straight line strategy ($r = 0.883$), the fast approach strategy ($r = 0.863$), and the slow approach strategy ($r = 0.580$).

TABLE B-4. CORRELATION COEFFICIENTS BETWEEN SUBJECT MEDIAN COSTS, MARK I COSTS, AND SELECTED ALTERNATIVE STRATEGIES

Costs	Subject Median	Mark I
Subject Median	1	0.916
Mark I	0.916	1
Composite	0.918	0.880
Absolute Minimum	0.909	0.892
Expected Minimum	0.907	0.943
Gradual Approach	0.883	0.824
Fast Approach	0.863	0.645
Slow Approach	0.580	0.727
Random, Without Penalty	0.384	0.421
Random, With Penalty	0.268	0.278

In terms of the statistical significance of the correlation coefficients and their differences, all strategies are statistically equivalent to the Mark I model with the exception of the slow approach strategy and the random strategy. Although the slow approach strategy and the random strategy without the miss-distance penalty have correlation coefficients which differ statistically from zero, the values are too low to be of practical interest. The random strategy which includes the miss-distance penalty yields a correlation coefficient which is statistically equal to zero.

Table B-4 also shows the correlation coefficients between Mark I costs and the cost associated with the eight strategies. The expected minimum strategy yields the highest correlation ($r = 0.943$). The statistical interpretations of these correlations generally agree with those obtained for the subject median costs. The strategies dependent on incremental costs again yield higher correlations than the remaining strategies. Those strategies independent of the incremental costs give the next highest set of correlation coefficients. Of these, the correlation coefficients for the fast approach and the slow approach differ statistically from 0.916. Again, the random strategies give small correlation coefficients. In general, these correlations indicate qualitative relations which would be expected.

Summary of Correlation Results for Individual Subjects

Table B-5 shows the correlation coefficients between individual subject costs for the 23 Mark I problems and the costs obtained from the Mark I model and the eight alternative strategies considered in the previous

TABLE B-5. CORRELATION COEFFICIENTS BETWEEN INDIVIDUAL SUBJECT COSTS, MARK I COSTS, AND SELECTED ALTERNATIVE STRATEGIES

Subject	Strategy*								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	0.11	0.06	0.77	0.35	0.79	0.87	0.78	0.88**	0.78
2	0.31	0.65	0.76	0.67	0.74	0.74	0.83**	0.75	0.81
3	0.23	0.42	0.64	0.76	0.83	0.81	0.87	0.81	0.92**
4	0.19	0.44	0.78	0.62	0.78	0.70	0.72	0.71	0.82**
5	0.13	0.12	0.80	0.39	0.82	0.85	0.79	0.85**	0.79
6	0.04	0.04	0.34	0.52	0.58**	0.49	0.55	0.49	0.55
7	0.24	0.43	0.95**	0.43	0.81	0.69	0.67	0.71	0.71
8	0.21	0.26	0.32	0.59	0.66	0.74	0.88**	0.73	0.71
9	0.11	0.45	0.64	0.77	0.93**	0.69	0.85	0.69	0.82
10	0.23	0.40	0.80	0.57	0.86**	0.83	0.86	0.83	0.85
11	0.17	0.01	0.77	0.22	0.71	0.80	0.67	0.81**	0.68
12	0.12	0.08	0.69	0.40	0.76	0.87	0.81	0.89**	0.82
13	0.39	0.78	0.81**	0.61	0.62	0.52	0.60	0.51	0.61
14	0.36	0.57	0.93**	0.49	0.74	0.70	0.69	0.70	0.74

*(1): Random with miss-distance penalty.
(2): Random without miss-distance penalty.
(3): Fast approach.
(4): Slow approach.
(5): Straight line approach.
(6): Absolute minimum.
(7): Expected minimum.
(8): Composite absolute and expected minimum.
(9): Mark I model.

** Denotes maximum correlation for given subject.

section. The maximum correlation obtained for each subject is indicated by a double asterisk. Examination of the table shows that the costs for subjects 7, 13, and 14 showed the highest correlation with the fast approach strategy. The costs associated with the straight line approach maximize the correlation coefficient for subjects 6, 9, and 10. The expected minimum strategy is represented by subjects 2 and 8. The composite strategy is most frequently occurring strategy, and best describes the costs yielded by subjects 1, 5, 11, and 12. Two subjects, 3 and 4, have costs that are best represented by the costs obtained from the Mark I model. The random strategies and the slow approach strategy are not represented by any of the subjects.

These results suggest the possibility of obtaining a classification of controllers according to the strategy showing the maximum correlation. Such an approach would require a considerable amount of experimental effort and a very careful specification of the set of alternative strategies.

APPENDIX C

INSTRUCTIONS TO SUBJECTS

APPENDIX C

INSTRUCTIONS TO SUBJECTS

Mark I

We have a contract with NASA to develop a mathematical model of how the human makes decisions in controlling a space vehicle. We are now at the point in the program where we have to collect some empirical data from the human to see how well our model describes the actual way he behaves. And this is why we have asked you to be a subject in the study.

In this study, your task will be to control the velocity of a vehicle while minimizing your fuel consumption. You will start at one velocity and have to change to another; after you reach the second velocity, you will have to change to a third, etc. So, your task is really one in which you have to control the vehicle through a sequence of different velocities and at the same time keep your fuel consumption as low as you can. This is similar to what an astronaut might do in checking out a series of satellites in space.

We have simulated the dynamics of the vehicle on the computer. For this flight there are just five controls available to you, -2, -1, 0, +1, and +2. The negative controls will always decrease your velocity, the postive controls will increase it, and the zero will leave it unchanged. In order to enter a -2 control you push the minus sign, then the 2, then the enter key. To enter a -1, you push the minus key, then the 1, then the enter. To enter a zero, you push the minus key, then the 1, then the enter. To enter a +1, you push the space bar, than a 1, then the enter. A +2 is entered by pushing the space bar, then the 2, then the enter.

Let's practice this a few times. Enter a +2. Now a -1. A 0. A +1. A -2.

Now, let's look at the first leg of the flight. Notice that four numbers are printed at the top of the page: Sub-Problem No., Initial Velocity, Final Velocity, and Number of Decisions. So, for this leg of the flight you'll begin at a velocity of 470 and want to go to a velocity of 590. You'll have 38 decisions, or in other words, 38 choices of the controls -2, -1, 0, +1, +2, in which to go from that initial velocity of 470 to the final velocity of 590.

Each of your decisions, or control choices, must be entered during a 5 second period when this light is on. Please don't enter your controls before the light comes on. If you fail to enter it before the light goes off, I'll enter your previous control for you.

Now let's look at the rest of the sheet. You'll notice there are six column headings printed on it. The first one, "Decision Number", just tells you how many decisions are left in this leg of the flight. So for this first leg of the flight, numbers in that column will run from 38 to 0.

The second column is headed "Current Velocity". This tells you how fast you're going. As you can see, "470" is printed in this column, indicating your present velocity.

The third column "Distance to Final Velocity", tells you how far your current velocity is from the final velocity. By the sign you also can tell whether you're above or below the final velocity. Since "470" is 120 units below 590, you'll notive that "-120" is printed in the column.

The fourth column "Control Value" shows you which one of the five controls you have chosen. So, -2, -1, 0, +1, or +2 will be printed here.

The fifth column is labeled "Fuel Cost". This indicates how much fuel was used on your last decision.

Finally, the sixth column is headed "Cumulative Fuel Cost". This column keeps a running sum of the fuel costs shown in column five. Remember, you're trying to keep your fuel cost as low as you can, so you'll want to pay close attention to these last two columns.

Now let's look at the right-hand position of the sheet. This section will print out a graphical display of your velocity along with the final velocity. Here's an example of what such a graph might look like after 10 decisions. You'll notice that the beginning velocity was 570 and the final velocity was 610. After the first decision, the velocity was 670, after the second it was 690, and so forth. The zero will always show the final velocity and the X's will indicate your present velocity.

There's one final bit of information we'd like to get from you as you maneuver through the flight. We'd like to get some idea about the way you're thinking about it. To do this we'd like you to imagine that I'm another astronaut waiting on the ground about to begin a similar flight, and that you're to radio back to me all information which you feel might be of some help in navigating such flights. In addition, I'd like you to state your confidence in the information you're radioing back. Let me give you a couple of examples of what such radioed-back statements might be. You might say, "I feel 90 percent certain that choosing all 2's is the best thing to do to conserve fuel". Or you might say, "I'm 95 percent certain that going to a straight line is not the best thing to do to conserve fuel".

Whenever possible, the statements you radio back should include something about fuel consumption since this is what you're trying to minimize. Remember, too, that these statements should be more in the form of advice about what I should do rather than just descriptions about what happened in your situation. For example, I think you can see that the two previous statements would be much more helpful to me than a statement like, "Boy, I really goofed that one." I'll ask for these statements at the end of each leg of the flight, but you can make them at any time they occur to you. Remember, I'll be beginning a similar flight but the trajectories and controls may be slightly different.

In order to radio your information back all you need to do is tell me and I'll turn on this tape recorder and record your statements.

Now if you should happen to miss the final velocity, you'll be penalized by having an additional fuel cost added to your cumulative fuel cost at the end of each leg of the flight. But even though you might miss a final velocity on one leg, you'll begin the next leg as though you had reached the goal. For example, suppose you wound up with a velocity of 580 after 38 decisions on this first leg instead of the desired final velocity of 590. Well, even if that happened you'd begin the second leg at 590 not 580. Of course, as I said, there'd be a penalty added to your fuel cost.

One final bit of caution before you begin. We've found that some subjects occasionally made an error such as pressing a +2 when they actually meant a -2. Therefore, be careful to depress the actual keys you want since the computer won't give you a second choice. One thing that should help you here is the sign shown in the third column, "Distance to Final Velocity". If that sign is negative, you'll know that you're below the final velocity and need a positive control in order to hit it. If no sign is present, you'll know that you're above the final velocity and, therefore, need a negative control.

So, briefly, what you're to do is control your vehicle through a series of different velocities by selecting a series of control values, -2, -1, 0, +1, +2. Each of these decisions should be entered when the light comes on. As you navigate the flight you're to radio back any information which you feel would be helpful to me in beginning a similar flight and you're to indicate your confidence in these statements. Lastly, you're trying to use as little fuel as possible during each of the legs of the flight.

Well, I think that's about it, do you have any questions?

O.K. Your first leg in the flight is going from a velocity of 470 to one of 590 in 38 decisions while minimizing fuel. You can begin by entering your first decision.

Questions to be Asked of Subjects

I. At end of each leg:

Do you have any advice for me in beginning a similar flight?

or

Any advice?

II. At the end of the problem:

Could you now summarize your advice or recommendations to me? Remember my flight won't be identical to yours, but it will be similar in many respects.

Mark II

We have a contract with NASA to develop a mathematical model of how the human makes decisions in controlling a space vehicle. We're now at the point in the program where we have to collect some empirical data from the human to see how well our model describes the actual way he behaves. This is why we've asked you to be a subject in the study.

In this study, your task will be to guide your vehicle through a series of points in space while minimizing fuel consumption. You will start at one point and have to guide your vehicle to another; after you reach the second point, you will have to go to a third; etc. So, your task is really one in which you have to control your vehicle through a sequence of different points in space and at the same time keep your fuel consumption as low as you can. This is similar to what an astronaut might do in checking out a series of satellites in space.

We've simulated the dynamics of the vehicle on the computer. For this flight there are just five controls available to you, -2, -1, 0, +1, and +2. In order to enter a -2 control, you push the minus key, then the 2, and then the enter. To enter a -1, you push the minus key, the 1 and then the enter. A zero is entered by pushing the space bar, the zero, and the enter. A +1 is entered by pushing the space bar, the 1, then the enter. And the +2 is entered by pushing the space bar, the 2, and the enter.

I know this is simple but let's practice a few times. Would you enter a +2 please? A -1. A 0. A +1. And a -2.

Now, let's look at the first leg of the flight. First, you'll notice that seven numbers are printed across the top of the page. These are: Trajectory No. 1--this means that you're on the first leg or trajectory of the flight. Second, Initial Position 1100.00--this is the position at which you're beginning the flight. Next, Final Position 1148.00--this is where you want to wind up at the end of this leg. Fourth, Initial Velocity 0.00. Fifth, Initial Acceleration 0.00. Sixth, Time Constant--Large. And finally seventh, Number of Decisions 20.

So, you're to go from a position 1100.00 to a position 1148.00 in 20 decisions while using as little fuel as you can.

Each of your decisions, or control choices, must be entered during a 5 second period when this light is on. Please don't enter your controls before the light comes on. If you fail to enter it before the light goes off, I'll enter your previous control for you.

Now let's look at the rest of the sheet. Eight column headings appear here. First, "Remaining Decisions"--since there are 20 decisions, numbers in this column will run from 20 down to 1. Next, "Control Choice"--your control, -2, -1, 0, +1, or +2, will appear here. The third column, "Current Position", tells you where you are after each decision. The fourth column gives your current velocity after each decision. In column five will appear your current accelerations. The next column, "Distance to Go", tells you how far your current position is from your final position. Since 1100.00 is 48.00 away from 1148.00, you'll notice that 48.00 is printed here. The seventh column is labeled "Fuel Cost" and indicates how much fuel was used on each decision. Finally, the eight column, "Cumulative Fuel Cost" keeps a running sum of the fuel costs shown in column seven. Remember, you're trying to use as little fuel as possible so you'll want to pay close attention to these last two columns.

There's one final bit of information we'd like to get from you as you maneuver through the flight. We'd like to get some idea about the way you're thinking about it. To do this we'd like you to imagine that I'm another astronaut waiting on the ground about to begin a similar flight, and that you're to radio back to me all information which you feel might be of some help in navigating such flights. In addition, I'd like you to state your confidence in the information you're radioing back. Let me give you a couple of examples of what such radioed-back statements might be. You might say, "I feel 90 percent certain that choosing all 2's is the best thing to do to conserve fuel".

Whenever possible, the statements you radio back should include something about feul consumption since this is what you're trying to minimize. Remember, too, that these statements should be more in the form of advice about what I should do rather than just descriptions about what happened in your situation. For example, I think you can see that the previous statement would be much more helpful to me than a statement like, "Boy, I really goofed that one." I'll ask for these statements at the end of each leg of the flight but you can make them at any time they occur to you. Remember, I'll be beginning a similar flight but the trajectories and controls may be slightly different.

In order to radio your information back all you need to do is to tell me and I'll turn on this tape recorder and record your statements.

You don't have to hit the exact final position at the end of each trajectory. If you're within $\pm$ 5 of the final position it's considered a "hit". So for this first leg if you end up between 1143 and 1153 you're considered on target.

If, however, you're further away than ± 5 you'll be penalized by having an additional fuel cost added to your score. But even though you might miss a final velocity on one leg, you'll begin the next leg as though you had reached the goal. For example, suppose you wound up at 1140 after 20 decisions on this first leg instead of the desired final position of 1148. Well, even if that happened you'd begin the second leg of 1148 not 1140. Of course, as I said, there'd be a penalty added to your fuel cost.

One final bit of caution before you begin. We've found that some subjects occasionally make an error such as pressing a -2 when they actually meant +2. Therefore, be careful to depress the actual keys you want. If you catch the error before you push the enter key, just tell me and I'll correct it.

So briefly what you're to do is control your vehicle through a sequence of points by selecting a series of control values, -2, -1, 0 +1, and +2. Each of these decisions should be entered when the light comes on. As you navigate through the flight you're to radio back any information which you feel would be helpful to me in beginning a similar flight and you're to indicate your confidence in these statements. Lastly, you're trying to use as little fuel as possible during each leg of the flight.

Well, that's about it. Do you have any questions?

O.K. Your first trajectory is going from 1100 to 1148 in 20 decisions while minimizing fuel. You can begin by entering your first decision.

<u>Questions to be Asked of Subjects</u>

I. At end of each leg:

Do you have any advice for me in beginning a similar flight?

or

Any advice?

II. At the end of the problem:

Could you now summarize your advice or recommendations to me? Remember my flight won't be identical to yours, but it will be similar in many respects.

APPENDIX D

VERBAL STATEMENTS MADE BY MARK I SUBJECTS FOR PROBLEMS 1, 12, AND 23

APPENDIX D

VERBAL STATEMENTS MADE BY MARK I SUBJECTS FOR PROBLEMS 1, 12, AND 23

This appendix consists of a listing of the verbal statements made by the Mark I subjects for Problems 1, 12, and 23. These problems may be taken as representative of the beginning, middle portion, and final portion of the Mark I problems.

Subject 1

Problem 1 — I found at higher speed you use less fuel for a given change of velocity and also that you use fuel even while maintaining same speed. Therefore it seems logical to go to the final velocity at the slowest possible rate so as to minimize fuel consumption.

Problem 12 — No statement. Statement from Problem 9--I found awhile ago that the characters 1 and 2 represent 10 and 20 miles per hour respectively. It is also wise to remember that the difference in velocities is negative or positive and the change in speed that you want is opposite. For example, if the difference is negative 50 your change should be positive in order to get closer to the final velocity.

Problem 23 — No statement. Statement from Problem 14--I wish to contradict my last statement. It is not necessarily true that the best operating velocity is in the same direction as the velocity which you are trying to obtain. I am 100 percent certain of this.

Subject 2

Problem 1 — At times I felt myself thinking about what I would do if the computer all of a sudden registered something other than 590. I caught myself thinking of what I would do if it appeared 480 or what I could do if it appeared 600 or 610 and I think this apprehension sort of made me pause too long in keeping it. In otherwords I didn't trust the computer. I thought it was trying to trick me.

Problem 12 I'm very positive (95%) there is always a point where you will consume very little, if any, fuel. I suggest that this point should be obtained as soon as possible and maintained as long as possible until it is necessary to start making jumps to either go up to or down to the final velocity. Maintain the velocity of fuel which accompanies the minimum amount of fuel for as long as possible before making the jump either way.

Problem 23 During the course of this flight in obtaining changes in velocity I recommend a minimal point of fuel consumption be located. Now this can be done by going either direction, from the initial velocity. For instance, should you want to increase your velocity you can either increase past your final velocity and then fall back increasing it fast to get to your minimal point or decrease below your initial velocity to reach your minimal point and then climb back up in the least amoung of steps. This findin the minimal point can be best accomplished by noting and remembering the early trends in decreasing and increasing velocity. For instance, on the first increase of the velocity, if you should increase it from past your final velocity and you find that on your first stop you consume a lot of fuel, then head in the other direction quickly and remember this.

Subject 3

Problem 1 No statement.

Problem 12 About 75 percent sure that you should play your hunches. I had a hunch on this one that I was going to have to go much higher than my final velocity and come back to it, but I didn't play my hunch and I ended up using extra fuel.

Problem 23 Use the first three steps to locate your points where the costs are equal to zero. Then continue along this point until you have the right number of steps to reach your final velocity in the least number of moves. I guess that's all. I think this is right. I say 90 percent confidence.

Subject 4

Problem 1 I would recommend coming within 50 miles of the final desired velocity, holding it within 50 miles per hour range, and holding it at zero, increase in velocity (which seems to spend the least amount of fuel) and then on the last three tried, attempt to bring it into the final desired velocity.

Problem 12 I'm 96 percent confident that you try to find a rest where there is no fuel consumed, hold it there, and then go into the final desired velocity at the end.

Problem 23 Try to find the trend in the fuel consumption and follow the trend and find out when it is decreasing and follow it until the minimal amount is used and hold it there as long as possible and go into the final desired velocity in the calculated number of tries at the end. 96 percent confident.

Subject 5

Problem 1 Seventy-five percent confident that you will go 20 miles per hours less than the desired final velocity and then hold it constant there. About 90 percent confident that you reach 20 miles per hour less when you final velocity, remain constant there, you will conserve fuel.

Problem 12 Once you have obtained the velocity at which the fuel consumption is least and come to the number of steps which it is required to reduce the velocity by one. Thereby you should reduce by 2 until you reach the final answer and hold constant there. With 85 percent confidence you will have minimum fuel consumption.

Problem 23 I feel if you find the velocity where the least fuel is being used you will be able to conserve fuel all during the flight. But the problem arises during the period at which you have to reach the final velocity due to the number of steps at the end. The fuel costs to reach those velocities at that time are much greater than they are earlier. So if you can find the point during the flight at which the fuel costs to reach the final velocity is the smallest and with no change; why I now feel 50 percent confident that this way would be the method in which to reach the least amount of fuel used.

Subject 6

Problem 1 — It's about 90 percent better to get to your goals first and then right on zero. It is cheaper that way. I also find that it is better to go to 2 to 1 very quickly over to your goal and then go down to zero.

Problem 12 — In this I found out it was advisable to go over the velocity, but that I missed it and the penalty is expensive. But once' again to keep the vost to a minimum, calculate the distance and make sure you hit it.

Problem 23 — I feel at the beginning that you should start out with a 2 or plus zero increase or decrease your velocity towards your goal. After you find your costs increase or decrease your velocity not so much to see if you are going in the right direction, and then try to get to your lowest cost and by doing this calculate how far you have to go out and at the last minute come in with increasing or decreasing your speed gradually toward your point. Ninety-five percent sure that going to you low cost or zero is the most inexpensive way and I also feel that 70 percent right that increasing your speed gradually at the end toward your end point or decreasing right at the end is the best.

Subject 7

Problem 1 — Accelerate to +2 about 550 and then level off to zero to about 10. You conserve fuel at that rate and then come in at a +1 about every two intervals until you come to three and then come straight in on your final velocity and hold that unitl your final point.

Problem 12 — No statement. Statement from Problem 11--I'm 90 percent sure that you should accelerate as fast as possible until reaching the meeting point and then level off until final trial.

Problem 23 — I'm about 75 percent certain that it would be best to fluctuate the velocity mid-way through the trials and then go directly to your rendezvous and hold back.

Subject 8

Problem 1 I suggest that you go below the final velocity speed as long as you can until you finish the number of trials before you take it up to the final velocity speed because you would use less fuel flying at a low speed. I'm confident with this right now based on results.

Problem 12 In deceleration move down -2 as fast as you can until you get to the zero fuel consumption and hold there as long as you can until you have to bring it up to the final velocity. Ninety percent confident.

Problem 23 I would recommend first you establish your fuel consumption at the present speed when you start then move toward your objective either + or - 1. When you have established that you are moving in the right or wrong direction correct toward zero fuel consumption. Hold at zero fuel consumption until as long as you can leaving enough decisions so that you can get to your final velocity. I'm 100 percent confident that this is the best way to save on fuel and reach the objective of the final velocity.

Subject 9

Problem 1 I think you should just keep moving around and try not to have any penalties and save money that way.

Problem 12 No statement. Statement from Problem 10--I'm confident 100 percent, I think I'm still following the same pattern. Make the slope very slight.

Problem 23 The way I orginally started was just practicing. About the first four sub-trials just to get the feel of things and finally I got the kind of pattern I want to follow. My pattern was not to move the vehicles so fast that to waste fuel in big jumps. What I did do was to go just nice and slow down the line, no big jumps. My confidence at the conclusion was a 100 percent.

Subject 10

Problem 1 — It helps not use the zero button at all, but to use a little of + like a +1 and then followed by a -1. 100 percent confident.

Problem 12 — No statement. Statement from Problem 9--After you determine your zero and are using the + zero to keep your costs at a minimum plan to use the minimum number of decisions to retain your final velocity like using the majority of +2 or -2 to get to that velocity once you determine the zero points. Confidence 100 percent.

Problem 23 — My strategies begin by first of all just using the 1, the lower increment and go away from the velocity that you are trying to attain. You do this in order to get a low point--a zero or a 2, 3. If you achieve 2, don't try to get any lower, just using the zero button maintain at that velocity as long as you can, and then right near the end using twos and ones in combinations maintain your final velocity. Confidence 100 percent.

Subject 11

Problem 1 — I found that I conserve most fuel by a reaching a velocity near the desired speed and then reducing it by or decreasing velocity by a minus number and then holding it constant which did not cost any fuel consumption. However, when I desired to bring this speed up to the desired velocity, I had trouble in reaching the same results. I'm about 50 percent sure that by reaching a zero fuel consumption and maintaining a constant velocity is the best way to conserve fuel.

Problem 12 — No statement. Statement from Problem 11--Theory has a few flaws in it. Number 1 your minimum fuel consumption might be a great distance or discrepancy from your desired goal, and the only way to reach the goal is to increase velocity. I have only a few penalties and each one has been a 1000. I'm not sure if that's a distance from your goral or if it is set at a 1000. I think it might be better to be out in left field than to take a 1000 penalty, I don't know how that would work. No other comment.

Problem 23 Felt the theory I proceeded under was 95 - 100 percent the best theory. The theory was to start out by decreasing or increasing your velocity to find out which direction would conserve the most fuel, and then proceeding in that direction until you reached your minimum fuel consumption, and maintain that period for as long as possible, taking note of the number of steps you would require to return to the desired velocity by the number at the left of your fuel consumption. It's either registered plus or minus and every 1 unit on the keyboard was worth 10 units of velocity. Therefore by comparing you could figure how many you needed to get back. In the first few problems I ruled out the possibility of reaching a speed and then hitting zero, because it didn't seem to increase or decrease fuel consumption any. However, in the last problem, I noticed that I was at 1200 units of fuel consumption and by pushing zero, it went to zero. I'm not sure what the reasoning behind this was. I still feel any method was the best way to approach the problem.

Subject 12

Problem 1 I think it is best to get up to point where your velocity is 570 and then remain at 570 because the fuel consumption if you do add on. At 570 there is zero fuel consumption and after that, wait until the last possible trial and then add 2 and you will reach the final velocity.

Problem 12 No statement. Statement from Problem 11--Be sure you know where you are at all times.

Problem 23 Well, first of all not each problem is alike and one must go into them experimentally and use what knowledge that has been gained from previous problems. This might help a little, I'm not saying it will. The best thing I can recommend is just to pick out the number with the least fuel consumption. Stay with this number until you reach a point where you have to go for the final velocity without receiving a penalty. Above all, do not receive a penalty because a penalty costs a lot more than the amount of fuel consumption approaching the penalty. I think it would be between 75 and 85 percent confident.

Subject 13

<u>Problem 1</u> When you press the first button you go up 10 at positive, when you press the second button you go up 20. There is penalty when you go back the first time. The second time there is and the third time there isn't. I believe this true from the record of the flight.

<u>Problem 12</u> I'm convinced that by going past the set speed and moving back you save a great deal of fuel consumption.

<u>Problem 23</u> I feel and have confidence in the fact that if you vary your speeds between raising and lowering them you will have a lower fuel consumption. When you get your lowest consumption at any time during your raising and lowering you keep it constant at that consumption. Either you can get the consumption or keep it at zero until the last minute when you have to raise it.

Subject 14

<u>Problem 1</u> Try not to accelerate too fast. Don't go at maximum acceleration when you first start the flight because fuel consumption seems to be relatively high. Remember to take off reasonably easy, don't go a full acceleration because the cost is prohibited. Fuel consumption is extremely high. From my own experience to this point I would say that I'm totally confident that it is going to run high if you accelerate and try to get the final velocity too quickly.

<u>Problem 12</u> I recommend that you accelerate with a control value of 2 until you pass the final velocity of 650 because running costs above 650 are considerably less than at 660. Also it seems the higher you go above 660 the less it costs to run. I would recommend going 20 at the most 40 miles above, watching carefully to make sure you can get back to the desired velocity within the number of decisions.

<u>Problem 23</u> I would recommend in general over the course of the experiment to first try to stay at the same velocity to see how much it is going to be there and if you have to go from a low to a high velocity try going toward the velocity. If it is higher

(if the cost is higher) approach the final velocity and then go back to your initial velocity until the last minute. Now the same holds true the other way around. Starting at a high initial velocity and you want to drop to a low one. First stay at your initial velocity, see what the cost is there, then drop toward your final velocity. Now if the cost is higher there than at your initial velocity, go back to your initial velocity until the last minute and then drop in and carefully avoid the penalty. I'm so confident in this if that I had to do this experiment all the time I would use that procedure. 100 percent confident.

APPENDIX E

ANALYSIS AND CLASSIFICATION OF VERBAL STATEMENTS

APPENDIX E

ANALYSIS AND CLASSIFICATION OF VERBAL STATEMENTS

The Mark I and Mark II models yield predicted verbal heuristics. The observed conditional probability that the heuristic of a subject will match the predicted heuristic is computed in the report (Pages 78-102; 112-114). In this appendix we consider those statements of the subject which do not match the predicted heuristics.

Table E-1 lists the heuristics of the subjects not contained in the list of possible heuristics generated by the Mark I model. The listed statements were extracted from statements selected by the panel of three judges described earlier. The statements were underlined by the judges as statements which, in their view, were heuristic statements not contained in the list of possible heuristics.

Shown in Table E-2 are some examples of statements which were also selected as possible heuristics by the judges. Because the statements do not serve as rules for making choices among the controls, these statements were classified as non-heuristic statements.

In this way each underlined statement extracted by a judge was described in one of the four categories: (1) a heuristic statement equivalent to that generated by the Mark I model, (2) a heuristic statement equivalent to one of those in the list of possible heuristics, but different from that generated by the Mark I model, (2) a heuristic statement differing from all those appearing in the list of possible heuristics, and (4) verbal statements which are non-heuristic.

Table E-3 shows the average number of subjects associated with each of the verbal statement categories for each trajectory. The average is obtained as the arithmetic mean of the number of statements assigned to a given category for each of the three judges. Trajectories 9, 11, 13, 20, 21, and 22 are exceptional because the model did not predict a heuristic for these trajectories. For these cases the number shown in column 2 gives the average number of subjects that state any one of the possible heuristics associated with the model. Column 4 shows the average number of subjects that stated a heuristic not contained in the model list. If this number is compared with the sum of the numbers given in Columns 2 and 3, it is seen that most of the heuristics stated by the subjects were contained in the model list.

Table E-1. Subject Heuristics Not Contained in List of Possible Heuristics Associated with Mark I Model

Subject	Trajectory	Heuristic
1	1	Go to final velocity at slowest possible rate.
	13	To minimize fuel, always move in the same direction.
5	11	Approach final velocity rapidly, observe minimum consumption point, allow for number of steps required to decrease velocity by 1, realize difference between by reducing by 1 and 2.
6	1	Get to goal early, then to zero.
	6	Cost increases at decreasing speeds.
7	23	Fluctuate velocity midway through trials, then go to rendezvous.
8	1	Stay below final velocity as long as possible.
	21	Until you have a rough idea of how much the fuel consumption will change, you ought to increase or decrease by only one level at a time rather than two.
9	1	Just keep moving around - try to avoid penalties.
	2	Move vehicle slowly.
	10	Make slope very slight.
	23	Go nice and slow - no big jumps.
10	1	Don't use the zero control.
	4	Approach final velocity but don't hit it until the end.
	5	Find zero consumption, and vary around it.
13	12	Exceed final velocity, then drop down.

Table E-1 (continued)

Subject	Trajectory	Heuristic
14	1	Try not to accelerate too fast - it costs too much.
	2	Don't be afraid to drop to final velocity, but don't drop below it.
	4	Use 2's to decrease to final velocity.

Table E-2. Examples of Non-Heuristic Statements Made by Mark I Subjects

Subject	Trajectory	Statement
1	12	A progression is formed when changing velocities.
2	1	Computer may trick subject.
	5	Calculate number of steps left to reach final velocity.
3	4	Watch careless mistakes.
	12	You should play your hunches.
	13	Don't be hesitant about playing hunches.
	14	Play your hunches.
	15	You should not play your hunches.
	16	Keep your eye on the final velocity - there's a pattern.
	20	You must find out which direction you should move.
6	5	Be sure where you are going.
	6	At decreasing speeds your cost increases.
	13	Cost is same between 530 and 550.
11	5	Be sure to end up on the initial velocity or get penalized.
	11	Be sure you know where you are at all times.
	21	Be sure not to make mistakes.
13	1	Strategy is based on immediate serial order.
	5	Unless you are going to final velocity, a certain consumption rate is inevitable.
	8	By using the "2" button, your velocity goes down quicker, sometimes.
	11	Each rocket is different, especially in fuel consumption.
	19	Changing velocities minimizes fuel consumption.
14	1	Full acceleration is extremely costly.

Table E-3. Average Number of Subjects Associated with Verbal Statement Categories

	Heuristic Statement			
	Contained in Model List			
Trajectory	Same as Used by Model	Different Than Used by Model	Not Contained in Model List	Non-Heuristic Statement
1	4 1/3	2 2/3	3	4
2	6 1/3	5 1/3	1 1/3	1
3	7 1/3	6	2/3	0
4	8	3	2 1/3	2/3
5	10	1 2/3	2	1/3
6	10	1 2/3	2	1/3
7	9 1/3	2 2/3	2	0
8	11 2/3	1 1/3	1	0
9	--	13	1	0
10	12 1/3	2/3	1	0
11	--	12 2/3	1 1/3	0
12	12	1	1	0
13	--	13	1	0
14	11 1/3	1 2/3	1	0
15	12	1	1	0
16	12	1	1	0
17	12	1	1	0
18	11 2/3	1 1/3	1	0
19	11 1/3	1 2/3	1	0
20	--	13	1	0
21	--	13	1	0
22	--	13	1	0
23	11 2/3	1/3	2	0

APPENDIX F

A COMPLETE PRINT-OUT OF RESULTS FOR PROBLEM NO. 10 FOR THE 14 MARK I SUBJECTS

APPENDIX F

A COMPLETE PRINT-OUT OF RESULTS FOR PROBLEM NO. 10 FOR THE 14 MARK I SUBJECTS

The following listing consists of the on-line print-out obtained for Problem 10 for the Mark I experiement. The six left-hand columsn show the meter eadings that resulted from the control values selected by the subjects. The headings of these columns denote the number of decisions remaining, current velocity, "distance" from final velocity, control value, incremental cost, and cumulative cost, respectively. The right side of the print-out consists of a graphical representation of the current velocity, denoted by X, and the desired final velocity, denoted by O. The penalty, if any, for missing the desired final velocity, and the final cumulative cost are shown at the end of the print-out for each subject.

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 1

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440 490 540 590 640 690 740 790 840
	630	-200		0	0	X O
16			2			
	650	-180		3200	3200	X O
15			2			
	670	-160		1800	5000	X O
14			2			
	690	-140		800	5800	X O
13			2			
	710	-120		200	6000	X O
12			2			
	730	-100		0	6000	X O
11			0			
	730	-100		0	6000	X O
10			0			
	730	-100		0	6000	X O
9			0			
	730	-100		0	6000	X O
8			0			
	730	-100		0	6000	X O
7			0			
	730	-100		0	6000	X O
6			0			
	730	-100		0	6000	X O
5			2			
	750	-80		200	6200	X O
4			2			
	770	-60		800	7000	X O
3			2			
	790	-40		1800	8800	X O
2			2			
	810	-20		3200	12000	X O
1			2			
	830	0		5000	17000	X

PENALTY = 0
FINAL CUMULATIVE COST = 17000
PAUSE OK

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 2

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	840
	630	-200		0	0					X				O
16			2											
	650	-180		3200	3200					X				O
15			2											
	670	-160		1800	5000						X			O
14			2											
	690	-140		800	5800						X			O
13			2											
	710	-120		200	6000							X		O
12			2											
	730	-100		0	6000							X		O
11			0											
	730	-100		0	6000							X		O
10			0											
	730	-100		0	6000							X		O
9			0											
	730	-100		0	6000							X		O
8			0											
	730	-100		0	6000							X		O
7			0											
	730	-100		0	6000							X		O
6			0											
	730	-100		0	6000							X		O
5			2											
	750	-80		200	6200							X		O
4			2											
	770	-60		800	7000								X	O
3			2											
	790	-40		1800	8800								X	O
2			2											
	810	-20		3200	12000									X O
1			2											
	830	0		5000	17000									X

PENALTY = 0
FINAL CUMULATIVE COST = 17000
PAUSE OK

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 3

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	840
	630	-200		0	0					X				O
16			2											
	650	-180		3200	3200					X				O
15			2											
	650	-180		3200	6400					X				O
14			2											
	670	-160		1800	8200						X			O
13			2											
	690	-140		800	9000						X			O
12			2											
	710	-120		200	9200						X			O
11			2											
	730	-100		0	9200							X		O
10			0											
	730	-100		0	9200							X		O
9			0											
	730	-100		0	9200							X		O
8			0											
	730	-100		0	9200							X		O
7			0											
	730	-100		0	9200							X		O
6			0											
	730	-100		0	9200							X		O
5			2											
	750	-80		200	9400							X		O
4			2											
	770	-60		800	10200							X		O
3			2											
	790	-40		1800	12000								X	O
2			2											
	810	-20		3200	15200								X	O
1			2											
	830	0		5000	20200									X

PENALTY = 0
FINAL CUMULATIVE COST = 20200
PAUSE OK

```
SUB-PROBLEM NO.  10
INITIAL VELOCITY -   630                    SUBJECT NO. 4
FINAL VELOCITY -   830
NO. OF DECISIONS -  16

 NO.  CUR.  DIST.  CV   COST   CUM.   440   490   540   590   640   690   740
DEC.  VEL.  F.V.                COST
      630   -200            0      0                       X
 16                 2
      650   -180         3200   3200                          X
 15                 2
      670   -160         1800   5000                             X
 14                 2
      690   -140          800   5800                                X
 13                 2
      710   -120          200   6000                                   X
 12                 2
      730   -100            0   6000                                      X
 11                0
    730   -100          0    6000                                    X
  10
             0
   730   -100        0   6000                                    X           O
  9          0
   730   -100        0   6000                                    X           O
  8          0
   730   -100        0   6000                                    X           O
  7          0
   730   -100        0   6000                                    X           O
  6          0
   730   -100        0   6000                                    X           O
  5          2
   750    -80      200   6200                                       X        O
  4          2
   770    -60      800   7000                                          X     O
  3          2
   790    -40     1800   8800                                             X  O
  2          2
   810    -20     3200  12000                                               X O
  1          2
   830      0     5000  17000                                                X
PENALTY =         0
FINAL CUMULATIVE COST =  17000
 PAUSE
         OK
```

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 5

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	840
	630	-200		0	0					X				O
16			2											
	650	-180		3200	3200					X				O
15			1											
	660	-170		2450	5650					X				O
14			2											
	680	-150		1250	6900						X			O
13			2											
	700	-130		450	7350						X			O
12			2											
	720	-110		50	7400							X		O
11			1											
	730	-100		0	7400							X		O
10			0											
	730	-100		0	7400							X		O
9			0											
	730	-100		0	7400							X		O
8			0											
	730	-100		0	7400							X		O
7			1											
	740	-90		50	7450							X		O
6			2											
	760	-70		450	7900							X		O
5			1											
	770	-60		800	8700								X	O
4			2											
	790	-40		1800	10500								X	O
3			1											
	800	-30		2450	12950								X	O
2			1											
	810	-20		3200	16150								X	O
1			2											
	830	0		5000	21150									X

PENALTY = 0
FINAL CUMULATIVE COST = 21150
PAUSE OK

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 6

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	840
	630	-200		0	0					X				O
16			0											
	630	-200		5000	5000					X				O
15			2											
	650	-180		3200	8200					X				O
14			2											
	670	-160		1800	10000						X			O
13			2											
	690	-140		800	10800						X			O
12			2											
	710	-120		200	11000						X			O
11			2											
	730	-100		0	11000							X		O
10			0											
	730	-100		0	11000							X		O
9			0											
	730	-100		0	11000							X		O
8			0											
	730	-100		0	11000							X		O
7			0											
	730	-100		0	11000							X		O
6			0											
	730	-100		0	11000							X		O
5			2											
	750	-80		200	11200							X		O
4			2											
	770	-60		800	12000							X		O
3			2											
	790	-40		1800	13800								X	O
2			2											
	810	-20		3200	17000								X	O
1			2											
	830	0		5000	22000									X

PENALTY = 0
FINAL CUMULATIVE COST = 22000
PAUSE OK

```
SUB-PROBLEM NO.  10
INITIAL VELOCITY -   630
FINAL VELOCITY -   830                                  SUBJECT NO. 7
NO. OF DECISIONS -  16

NO.  CUR. DIST. CV   COST  CUM.  440   490   540   590   640   690   740   790   840
DEC. VEL.  F.V.            COST
     630  -200           0      0                       X                        O
16               2
     650  -180        3200   3200                         X                      O
15               2
     670  -160        1800   5000                            X                   O
14               2
     690  -140         800   5800                              X                 O
13               2
     710  -120         200   6000                                X               O
12               2
     730  -100           0   6000                                   X            O
11               2
     750   -80         200   6200                                     X          O
10               0
     750   -80         200   6400                                     X          O
9                2
     770   -60         800   7200                                        X       O
8                2
     790   -40        1800   9000                                          X     O
7                0
     790   -40        1800  10800                                          X     O
6                2
     810   -20        3200  14000                                            X   O
5                2
     830     0        5000  19000                                               X
4                0
     830     0        5000  24000                                               X
3                0
     830     0        5000  29000                                               X
2                0
     830     0        5000  34000                                               X
1                0
     830     0        5000  39000                                               X

PENALTY =         0
FINAL CUMULATIVE COST =  39000
 PAUSE          OK
```

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 8

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	840
	630	-200		0	0									
16			0							X				O
	630	-200		5000	5000									
15			-2							X				O
	610	-220		7200	12200									
14			2						X					O
	630	-200		5000	17200									
13			2							X				O
	650	-180		3200	20400									
12			2							X				O
	670	-160		1800	22200									
11			2								X			O
	690	-140		800	23000									
10 2			+'								X			O
	710	-120		200	23200									
9			1								X			O
	720	-110		50	23250									
8			2									X		O
	740	-90		50	23300									
7			2									X		O
	760	-70		450	23750									
6			0									X		O
	760	-70		450	24200									
5			2									X		O
	780	-50		1250	25450									
4			2										X	O
	800	-30		2450	27900									
3			1										X	O
	810	-20		3200	31100									
2			2										X	O
	830	0		5000	36100									
1			0											X
	830	0		5000	41100									X

PENALTY = 0
FINAL CUMULATIVE COST = 41100
PAUSE OK

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 9

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	8
	630	-200		0	0					X				O
16			1											
	640	-190		4050	4050					X				O
15			1											
	650	-180		3200	7250					X				C
14			0											
	650	-180		3200	10450					X				C
13			1											
	660	-170		2450	12900					X				C
12			1											
	670	-160		1800	14700					X				C
11			2											
	690	-140		800	15500						X			(
10			1											
	700	-130		450	15950						X			C
9			2											
	720	-110		50	16000						X			(
8			2											
	740	-90		50	16050							X		(
7			2											
	760	-70		450	16500							X		(
6			1											
	770	-60		800	17300							X		(
5			1											
	780	-50		1250	18550							X		(
4			1											
	790	-40		1800	20350								X	(
3			1											
	800	-30		2450	22800								X	(
2			2											
	820	-10		4050	26850								X'	
1			1											
	830	0		5000	31850									

PENALTY = 0
FINAL CUMULATIVE COST = 31850
PAUSE
OK

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 10

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	840
	630	-200		0	0									
16			1							X				O
	640	-190		4050	4050									
15			2							X				O
	660	-170		2450	6500									
14			2							X				O
	680	-150		1250	7750									
13			2								X			O
	700	-130		450	8200									
12			2								X			O
	720	-110		50	8250									
11			2									X		O
	740	-90		50	8300									
10			1									X		O
	750	-80		200	8500									
9			2									X		O
	770	-60		800	9300									
8			2										X	O
	790	-40		1800	11100									
7			-2										X	O
	770	-60		800	11900									
6			0										X	O
	770	-60		800	12700									
5			0										X	O
	770	-60		800	13500									
4			0										X	O
	770	-60		800	14300									
3			2										X	O
	790	-40		1800	16100									
2			2										X	O
	810	-20		3200	19300									
1			2										X	O
	830	0		5000	24300									
														X

PENALTY = 0
FINAL CUMULATIVE COST = 24300
PAUSE OK

```
SUB-PROBLEM NO.  10
INITIAL VELOCITY -   630
FINAL VELOCITY -   830                                SUBJECT NO. 11
NO. OF DECISIONS -  16

 NO. CUR.  DIST.  CV  COST   CUM.   440  490  540  590  640  690  740  790  840
DEC. VEL.  F.V.                COST
      630  -200         0      0                     X                   O
  16                2
      650  -180      3200   3200                       X                 O
  15                2
      670  -160      1800   5000                         X               O
  14                2
      690  -140       800   5800                           X             O
  13                2
      710  -120       200   6000                             X           O
  12                1
      720  -110        50   6050                              X          O
  11                1
      730  -100         0   6050                               X         O
  10                1
      740   -90        50   6100                                X        O
   9                1
      750   -80       200   6300                                 X       O
   8               -2
      730  -100         0   6300                               X         O
   7                2
      750   -80       200   6500                                 X       O
   6                2
      770   -60       800   7300                                   X     O
   5                2
      790   -40      1800   9100                                     X   O
   4               -2
      770   -60       800   9900                                   X     O
   3                2
      790   -40      1800  11700                                     X   O
   2                2
      810   -20      3200  14900                                       X O
   1                2
      830     0      5000  19900                                         X

PENALTY =          0
FINAL CUMULATIVE COST =  19900
 PAUSE          OK
```

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 12

NO. DEC.	CUR. VEL.	DIST. F.V.	CV	COST	CUM. COST	440	490	540	590	640	690	740	790	84
	630	-200		0	0					X				O
16			2											
	650	-180		3200	3200					X				O
15			2											
	670	-160		1800	5000						X			O
14			2											
	690	-140		800	5800						X			O
13			2											
	710	-120		200	6000						X			O
12			2											
	730	-100		0	6000							X		O
11			2											
	750	-80		200	6200							X		O
10			0											
	750	-80		200	6400							X		O
9			0											
	750	-80		200	6600							X		O
8			2											
	770	-60		800	7400							X		O
7			-2											
	750	-80		200	7600							X		O
6			0											
	750	-80		200	7800							X		O
5			0											
	750	-80		200	8000							X		O
4			2											
	770	-60		800	8800							X		O
3			2											
	790	-40		1800	10600								X	O
2			2											
	810	-20		3200	13800								X	O
1			2											
	830	0		5000	18800									X

PENALTY = 0
FINAL CUMULATIVE COST = 18800
PAUSE

```
SUB-PROBLEM NO.  10
INITIAL VELOCITY -   630
FINAL VELOCITY -   830
NO. OF DECISIONS -  16                                    SUBJECT NO. 13

NO.  CUR.  DIST.  CV   COST   CUM.   440  490  540  590  640  690  740  790  8
DEC. VEL.  F.V.               COST
     630   -200           0      0                         X                   C
16                0
     630   -200        5000   5000                         X                   C
15                2
     650   -180        3200   8200                           X                 C
14                2
     670   -160        1800  10000                             X               C
13                2
     690   -140         800  10800                               X             C
12                2
     710   -120         200  11000                                 X           C
11                2
     730   -100           0  11000                                   X         C
10                0
     730   -100           0  11000                                   X         C
9                 0
     730   -100           0  11000                                   X
8                 0
     730   -100           0  11000                                   X
7                 0
     730   -100           0  11000                                   X
6                 0
     730   -100           0  11000                                   X
5                 2
     750    -80         200  11200                                     X
4                 2
     770    -60         800  12000                                       X
3                 2
     790    -40        1800  13800                                         X
2                 2
     810    -20        3200  17000                                           X
1                 2
     830      0        5000  22000

PENALTY =          0
FINAL CUMULATIVE COST =   22000
 PAUSE          OK
```

SUB-PROBLEM NO. 10
INITIAL VELOCITY - 630
FINAL VELOCITY - 830
NO. OF DECISIONS - 16

SUBJECT NO. 14

```
 NO. CUR. DIST. CV  COST  CUM.  440  490  540  590  640  690  740  790  840
DEC. VEL.  F.V.           COST
      630  -200        0      0                     X                   O

 16              2

      650  -180     3200   3200                       X                 O

 15              2

      670  -160     1800   5000                         X               O

 14              1

      680  -150     1250   6250                          X              O

 13              2

      700  -130      450   6700                            X            O

 12              2

      720  -110       50   6750                              X          O

 11              2

      740   -90       50   6800                                X        O

 10              2

      760   -70      450   7250                                  X      O

  9              2

      780   -50     1250   8500                                    X    O

  8              1

      790   -40     1800  10300                                     X   O

  7              2

      810   -20     3200  13500                                       X O

  6              0

      810   -20     3200  16700                                       X O

  5              0

      810   -20     3200  19900                                       X O

  4              2

      830     0     5000  24900                                         X

  3              0

      830     0     5000  29900                                         X

  2              0

      830     0     5000  34900                                         X

  1              0

      830     0     5000  39900                                         X
```

PENALTY = 0
FINAL CUMULATIVE COST = 39900
PAUSE OK